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THE NULL RATIO CHANGE THEOREM IN LIFE TESTING

ABSTRACT

In research and development programs which have life (reliability) improvement as their goals, it is important to have a fundamental grasp of the principles involved in decisions about the extent of improvement, as well as the index of confidence associated with any improvement of a specified amount (null ratio) . The author discusses the general theory of such improvements with their associated confidence indices in those cases where Weibull analysis is the basis of the statistical decisions in life test programs. As a result of all this elegant unification of ideas by means of the NULL RATIO CHANGE THEOREM , the test engineer's decision job in reliability programs aimed at life improvements is made straightforward and without the usual messy confusion of what's what as far as promised improvements are concerned .

The Null Ratio Change Theorem in Life Testing

PURPOSE:

The purpose of the Null Ratio Change Theorem is to make it possible to determine what CONFIDENCE can be assigned to LIFE RATIO which is between UNITY and the OBSERVED SAMPLE LIFE RATIO in a comparison test of two Weibull plots.

DEFINITIONS AND SYMBOLISM:

(A) The POPULATION LIFE RATIO for which we desire a CONFIDENCE INDEX is called the NULL RATIO.

(B) The SAMPLE LIFE RATIO between corresponding points (at equal quantiles) in the two Weibull plots is called the OBSERVED LIFE RATIO.

(C) Given an OBSERVED LIFE RATIO ρ, we denote the CONFIDENCE we have in a NULL RATIO x by the symbol

$$C(x, \rho) .$$

(D) We call $C(1, \rho)$ the SIGNIFICANCE LEVEL (i.e., the CONFIDENCE of at least a UNIT POPULATION LIFE RATIO) when we observe a SAMPLE LIFE RATIO of ρ between two Weibull plots.

THE NULL RATIO CHANGE THEOREM:

The confidence for a null ratio x_1 as derived from an OBSERVED LIFE RATIO (for a given Weibull slope and a given total d.f.) is equal to the confidence for a null ratio x_2 for an OBSERVED SAMPLE LIFE RATIO

$$\left(\frac{x_2}{x_1}\right) \rho ,$$ for the same Weibull slope and total d.f.

Symbolically, we write this NULL RATIO CHANGE THEOREM as follows:

$$C(x_1, \rho) = C\left[x_2, (x_2/x_1)\rho\right] \qquad (1)$$

COROLLARY TO THE NULL RATIO CHANGE THEOREM:

Putting $x_2 = 1$ in (1):

$$C(x_1, \rho) = C(1, \rho/x_1) \qquad (2)$$

Equation (2) tells us that if we desire the confidence associated with a NULL RATIO x_1 for an OBSERVED SAMPLE LIFE RATIO ρ, we simply determine the SIGNIFICANCE INDEX for an OBSERVED LIFE RATIO (ρ/x_1) (assuming the same Weibull slope and total d.f.).

NUMERICAL EXAMPLE:

Suppose we have two neighboring Weibull plots, each of Weibull Slope 1.5. Suppose the left hand plot has sample size 7, and suppose the right hand Weibull Plot has sample size 10. If the OBSERVED SAMPLE MEAN LIFE RATIO is 1.75, what is the CONFIDENCE that the ACTUAL POPULATION MEAN LIFE RATIO is at least 1.2 ?

SOLUTION:

The formula for the SIGNIFICANCE of an OBSERVED SAMPLE MEAN LIFE RATIO ρ, given a WEIBULL SLOPE b, and TOTAL d. f. T is

$$C = 1 - 1/2\,\rho^{-bT^{1/4}} = C(1, \rho)$$

In this example: $\rho = 1.75$; $b = 1.5$

$$T = 6 \times 9 = 54$$

Hence, the SIGNIFICANCE LEVEL (i.e., Confidence of at least a UNIT POPULATION MEAN LIFE RATIO) is

$$C = C(1, 1.75) = 1 - 1/2(1.75)^{-1.5(54)^{1/4}}$$

$$C = \underline{.9486}$$

However, the CONFIDENCE OF AT LEAST A 1.2 POPULATION MEAN LIFE RATIO is

$$C(1.2, 1.75) = C(1, 1.75/1.2) = C(1, 1.45833)$$

$$= 1 - 1/2(1.45833)^{-1.5(54)^{1/4}} = \underline{.8922} \quad \text{(ans.)}$$

The confidence to be associated with the OBSERVED SAMPLE LIFE RATIO is always 50% .

The reason why we assign 50% confidence to the observed sample life ratio is due to the fact that the sample Weibull plots are based on Median Ranks .

The SIGNIFICANCE FORMULA for an OBSERVED SAMPLE $B_{10\%}$ LIFE RATIO of magnitude ρ (assuming a Weibull slope b and TOTAL DEGREES of FREEDOM T) is

$$C = C(1, \rho) = \frac{1}{1 + \rho^{-bT^{\frac{1}{4}}\sqrt{.2}}} \qquad (3)$$

The test data in the numerical example yield a confidence index of .7345 at the $B_{10\%}$ level (use the FORMULA (3) above .)
(this is the significance index for a UNIT NULL RATIO .)

According to those same test data , we can promise with CONFIDENCE .6651 that the POPULATION $B_{10\%}$ LIFE RATIO will be at least 1.2 .
(Use the NULL RATIO CHANGE THEOREM.)