Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
P.O. BOX 36504 • GROSSE POINTE, MICHIGAN 48236 • (313) 886-7976

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR
Volume 15
Bulletin 2

May , 1985

THE SCIENCE OF PRODUCT ASSURANCE TESTING MANAGEMENT

THE BASIC ELEMENTS OF PRODUCT ASSURANCE TESTING

Product Assurance Testing Management is a science with three basic elements. These are :

I : Performance Goal (s) (i. e., targets) for a product at various conditions .

II : Actual Tests on the product at various conditions , including field tests .

III : A Confidence Index for the product based on a comparison (i. e., gathered evidence) of the test data versus the performance goals .

Once the accumulated evidence and its resultant confidence index have been determined from a series of tests compared to their goals , it is necessary to decide whether or not the resultant confidence is sufficient to comply with what is known as "The Dollar Basis of Confidence" . This is explained in the next section .

THE DOLLAR BASIS OF CONFIDENCE

A product failing to meet a guaranteed goal (such as a warranty promise) will cause a dollar loss (say $L) .

A product which successfully operates to a guaranteed goal (such as a warranty promise) will create a dollar gain (or profit) (say $G) .

The Ratio $\dfrac{\text{Dollar Loss Due to Failure to Meet Goal}}{\text{Dollar Gain Due to Success in Meeting Goal}}$

is what is known as "THE ODDS REQUIRED TO BREAK EVEN"

Thus ,

$$\text{ODDS REQUIRED TO BREAK EVEN} = \frac{L}{G} \tag{1}$$

denotes this Odds by θ_1 . Then $\theta_1 = \frac{L}{G}$.

But , in general , in manufacturing and selling a consumer product we want to realize a net profit (not just brak even) . So, the odds in our favor (of being profitable) must be more than (L/G) . Let us say we want to make K times as much money from successful items as we ever lose from failed items (i. e. , those which fail to meet the promised warranty) .

Then it follows that the necessary odds in favor of success vs. failure becomes

$$\theta_K = \frac{KL}{G} \tag{2}$$

This number θ_K represents the odds we need from our testing program in favor of the product being able to live up to the goal (i. e. , warranty promise or target) .

THE RELATIONSHIP BETWEEN ODDS AND MATHEMATICAL CONFIDENCE

Mathematically we define confidence as follows :

$$\text{CONFIDENCE} = \frac{\text{ODDS}}{1 + \text{ODDS}} \qquad (3)$$

This is because mathematically odds are defined as follows :

$$\text{ODDS} = \frac{\text{CONFIDENCE OF SUCCESS}}{\text{CONFIDENCE OF FAILURE}} .$$

But , Confidence of Success + Confidence of Failure = 1 .

So, Confidence of Failure = 1 - Confidence of Success .

So,
$$\text{ODDS} = \frac{\text{CONFIDENCE OF SUCCESS}}{1 - \text{CONFIDENCE OF SUCCESS}}$$

If we simply use the word CONFIDENCE to denote <u>Confidence of Success</u> ,

then
$$\text{ODDS} = \frac{\text{CONFIDENCE}}{1 - \text{CONFIDENCE}} .$$

Solving this relation for confidence we obtain

$$\text{CONFIDENCE} = \frac{\text{ODDS}}{1 + \text{ODDS}} ,$$ which is Equation (3) .

TESTING REQUIREMENTS AS DICTATED BY DOLLARS LOST VS. DOLLARS GAINED

According to Equation (2) , the odds required to gain K times as much as we lose in the long run , the odds in favor of success in meeting a goal (warranty promise) must be

$$\sigma_K = \frac{KL}{G} \qquad \text{(same as Eq. (2))} \tag{4}$$

Where

L = Dollars Lost due to each failure to meet the reliability goal .

G = Dollars Gained from each success in meeting the reliability goal .

K = Profitability Factor , which represents the desired long run advantage ratio of (Dollar Gained/Dollars Lost) after counting up all failed cases and all sucessful cases .

Since confidence (probability of success for reliability promise) is defined to be (Odds/1 + Odds) , we can state that the REQUIRED CONFIDENCE

is

$$C_K = \frac{\left(\frac{KL}{G}\right)}{1 + \left(\frac{KL}{G}\right)} \tag{5}$$

Hence, the test sample must be large enough to give this confidence C_K of meeting the reliability goal .

NUMERICAL EXAMPLE OF SCIENTIFIC PRODUCT ASSURANCE TESTING

Suppose that the desired goal line on Weibull paper for a certain product is such that

Goal Line Weibull slope = b = 2
Goal Line Characteristic Life = 1000 hrs. = θ

Suppose , furthermore , that if the product turns out inferior to this goal line we lose $10, 000, 000 if the product is at least as good as the goal line. Now suppose we test 5 items with the following results :

Item #1 ran for x_1 = 800 hrs. (unfailed)
Item #2 ran for x_2 = 300 hrs. (unfailed)
Item #3 ran for x_3 = 1050 hrs. (failed)
Item #4 ran for x_4 = 900 hrs. (failed)
Item #5 ran for x_5 = 1100 hrs. (unfailed)

QUESTION : Do these test results yield enough confidence to realize twice as much in long run gains as long run losses ?

SOLUTION

We have L = $10, 000, 000 ; G = $1, 000, 000 ; K = 2

Hence , the Required Odds are

$$O_K = KL/G = 2(10,000,000)/1,000,000 = 20 .$$

The Required Confidence is , therefore ,

$$C_K = 20/1 + 20 = 20/21 = .95238 .$$

From the test results the Entropy Total on the Goal Line is

$$\mathcal{E}_{total} = (x_1/\theta)^b + (x_2/\theta)^b + (x_3/\theta)^b + (x_4/\theta)^b + (x_5/\theta)^b$$

$$= \left(\frac{800}{1000}\right)^2 + \left(\frac{300}{1000}\right)^2 + \left(\frac{1050}{1000}\right)^2 + \left(\frac{900}{1000}\right)^2 + \left(\frac{1100}{1000}\right)^2$$

$$= 3.8525$$

Since there are 2 failures in the list , it follows that the Average Entropy per Failure is

$$\mathcal{E}_{ave} = \mathcal{E}_{total}/2 = 3.8525/2 = 1.92625 .$$

Therefor , the Evidence in favor of meeting the goal is

$$E = \frac{\pi}{\sqrt{3}} \sqrt{2}\,(1.92625 - 1) = 2.37593 .$$

The Confidence from the test results (of meeting the goal) is

$$c = \frac{1}{1 + e^{-E}} = \frac{1}{1 + e^{-2.37593}} = .91497 .$$

Since .91497 is less than .95238 , this test provides INSUFFICIENT evidence of meeting the goal . More testing is needed in this case .

CONCLUSION

The science of product assurance testing management is easily applied to any testing situation by remembering two basic ideas :

(I) : The required confidence of meeting a specified goal is determined by the dollar expense caused by failure to meet the goal in comparison to the dollar gain from success in meeting the goal , as well as the desired profitability factor in the long run .

(II) : The amount of testing done (sample sizes and running times) must be sufficient to yield at least the same confidence index dictated by (I) above .