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GUIDING PRINCIPLES IN PROTOTYPE TESTING

INTRODUCTION

In any testing program for product life or reliability there are basic guiding principles which form the basis for designing and evaluating experimental testing , as well as routine control testing . This is particularly true of prototype tests . In this bulletin we give five guiding principles and some theorems relating to these principles in the case of prototype tests .

PRINCIPLE # 1 : Know what your reliability goal line is on Weibull paper .

PRINCIPLE # 2 : Know what the Entropy Growth Curve of your prototype machine looks like with respect to the reliability goal line .

PRINCIPLE # 3 : Learn how to analyze repeated failures together with repairs and renewals .

PRINCIPLE # 4 : Learn how to assign and analyze redesign credit numbers.

PRINCIPLE # 5 : In order to predict future failures on any part which is replaced or repaired you must know to what extent the renewal is debited .

DISCUSSION OF PRINCIPLE # 1

DEFINITION : A durability goal line on Weibull paper is a line or curve which you want your product to be at least as good as .

For example , in Figure 1 below , the curve AB represents a goal line for the life distribution of a machine in servce . Because of this desired goal the designer would like to have his machine always display longer life at any stage in its history than that indicated by the goal line .
In other words , a well designed machine keeps to the right of the goal line.

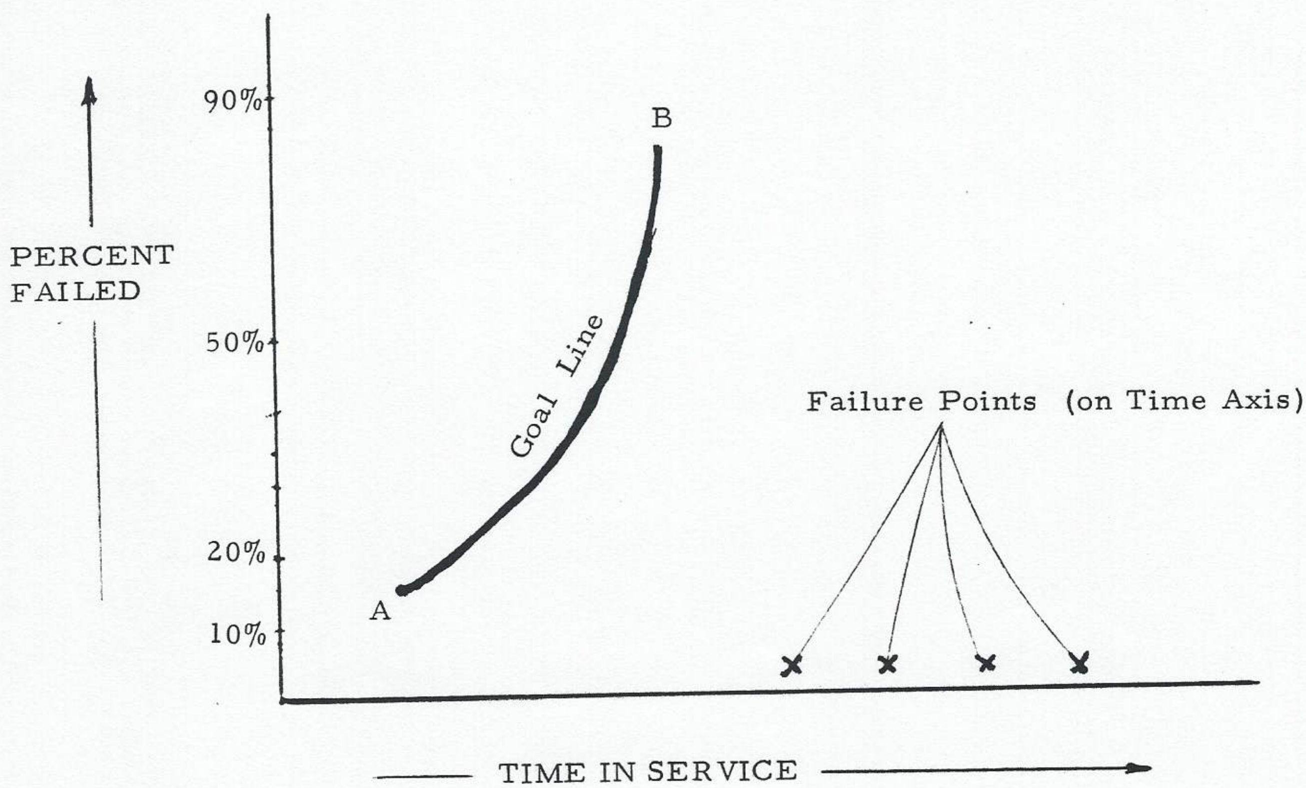

FIGURE 1

DISCUSSION OF PRINCIPLE #2

DEFINITION OF ENTROPY : The Entropy level in a distribution is defined at each point in the distribution as follows :

If the fraction F of the distribution has been accounted for (i. e. , the quantile F has been reached) , then the Entropy at that stage is

$$\text{ENTROPY} = \mathcal{E} = \ln\left(\frac{1}{1 - F}\right) .$$

In a two-parameter Weibull distribution defined by

$$F(x) = 1 - e^{-(x/\theta)^b}$$

Where x = service time

$F(x)$ = Fraction of machines failed at least once in service time x.

b = Weibull slope

θ = Characterisitic life ,

The Entropy at service time x becomes $\mathcal{E}(x) = \ln \frac{1}{1 - F(x)} = (x/\theta)^b$.

The superiority of a prototype machine to a goal line is determined by the magnitude of the AVERAGE ENTROPY PER FAILURE within the goal line by using the goal line's Entropy function on each failure occurrance in the prptotype machine .

DISCUSSION OF PRINCIPLE #3

BASIC FACT OF LIFE : A prototype machine will experience repeated failures, while a Weibull plot is made up of first failure only .

SO THE QUESTION IS : How can repeated failures be factored into Weibull parameter determinations ?

ANSWER : By using an Entropy Growth Curve obtained by plotting Failures per Machine versus service time on log-log paper .

This is because ---- Failure per Machine in accumulated time x is the Entropy in accumulated time x .

Furthermore if the life distribution of the machine is a two-parameter Weibull distribution , it follows that

I : The slope of the log-log plot is the Weibull slope .

II : The time to Unity Entropy is the characteristic life .

DISCUSSION OF PRINCIPLE # 4

DEFINITION : A redesign credit number for an improved replacement part in a system is apositive number p between 0 to 1 which represents the survival probability of the part for the same service time at which the part's predecessor failed .

Thus , if the predecessor failed at service time x_o (zero Survival probability) now the improved part has survival probability p for the same service time x_o .

In general , this means that the part is improved to the extent that if its original cumulative distribution function of failures had a value $F(x_o)$ at service time x_o , now the improved part at service time x_o has a cumulative distribution function $G(x)$, such that at x_o

$$G(x_o) = (1 - p)\,F(x_o) ,$$ where

p = redesign credit number .

DISCUSSSION OF PRINCIPLE # 5

DEFINITION : A replacement part is said to be fully debited if at the time of replacement it takes on the same age as the system into which it goes .

If the part is fully non-debited it starts up with age zero at the time of replacing its predecessor .

If the part is partially debited it starts up with an age somewhere between age zero and the system's total accumulated age at the time of replacement .

The extent to which a replacement part is debited affects predictions of future failures of the part . For this reason it is imperative to know to what extent the replacement part is debited .

MISCELLANEOUS THEOREMS RELATED TO THE GUIDING PRINCIPLES IN PROTOTYPE TESTING

THEOREM ON GOAL LINE ENTROPY :

We can relate principle #1 and principle #2 with the following theorem:

Given a set of k prototype machines with the following histories :

$$\left.\begin{array}{l} \text{Machine \#1 has run } x_1 \text{ hours and failed } r_1 \text{ times .} \\ \text{Machine \#2 has run } x_2 \text{ hours and failed } r_2 \text{ times .} \\ \quad\vdots \\ \text{Machine \#k has run } x_k \text{ hours and failed } r_k \text{ times .} \end{array}\right\}$$

Then the average Entropy per failure for these machines in a Goal Line defined by the cumulative distribution function of failure $F(x)$ is

$$\mathcal{E}_{\text{ave.}} = \frac{\ln \dfrac{1}{1 - F(x_1)} + \ln \dfrac{1}{1 - F(x_2)} + \ldots + \ln \dfrac{1}{1 - F(x_k)}}{r_1 + r_2 + \ldots + r_k}$$

The Evidence that the machines are at least as good as the goal line is then

$$E = \frac{\pi}{\sqrt{3}}\left(\mathcal{E}_{\text{ave.}} - 1\right)\sqrt{r_1 + r_2 + \ldots + r_k}$$

This Evidence implies a confidence of being at least as good as the goal line defined by

$$C = \frac{1}{1 + e^{-E}} .$$

THEOREMS ON REDESIGN CREDIT NUMBERS

FIRST REDESIGN CREDIT THEOREM :

The log-log plot of Entropy Growth versus service time before redesign credits can be modified by counting each redesigned part failure as only the fraction $q = 1 - p$ of a failure instead of a single failure wherever the original part failed on the service time axis .

SECOND REDESIGN CREDIT THEOREM :

If a part failing at service time x_o has Weibull parameters b and θ before redesign , then after a redesign with credit number p at the same time x_o , the new Weibull parameters of the improved part alone will be b and $\hat{\theta}$, where the Weibull slope b is unchanged and

$$\hat{\theta} = \frac{x_o}{\left[\ln \frac{1}{p + q e^{-(x_o/\theta)^b}} \right]^{1/b}}$$

Where $q = 1 - p$

CONCLUSION

We have listed the five guiding principles of prototype testing and explained their meanings it can be seen that these principles are very fundamental and form the logical basis for a systematic approach in prototype testing , especially when we make changes for the improvement of a product's durability during the prototype stage of development .