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MAKING ENTROPY PLOTS CONSISTENT WITH MEDIAN RANKS BY EMPLOYING BENARD TYPE EMPIRICAL MODIFICATIONS

INTRODUCTION

The handy Entropy Method of estimating Weibull parameters from any data set consisting of both failed and suspended items is based on the concept of AVERAGE or MEAN RATES OF FAILURE in each interval between failures , as well as employing the NUMBER OF ACTIVE ITEMS in such intervals. Then , for each interval , we form the quotient

$$\left(\frac{\text{Number Failed}}{\text{Number Active}}\right) \qquad (1)$$

To represent the AVERAGE FAILURE RATE PER ACTIVE ITEM . Because such average failure rates (i. e. , Entropy Increments) are employed in each interval between failures we find that the Weibull parameters thus arrived at DO NOT AGREE with MEDIAN RANK plots of the same data.

In order to correct this situation it becomes necessary to modify the Quotient (1) shown above in much the same manner as the order statistic number and sample size are modified in generating BENARD'S FORMULA for the Median rank of the j^{th} order statistic in N , i. e., we change

$$j/n \text{ into } (j - .3/N + .4)$$

The details of these modifications in the case of Entropy plots are discussed in this bulletin. It should be remembered that the proposed modifications are EMPIRICAL and , therefore , ONLY APPROXIMATIONS TO MEDIAN FAILURE RATES .

EXAMPLE ILLUSTRATING THE MODIFIED PROCEDURE

Suppose we had the following data on some failures in a fleet of motor vehicles :

Vehicle No.	Mileage Reached	Failed or Unfailed
1	20,000	S
2	30,000	← F
3	35,200	S
4	39,010	S
5	46,005	← F
6	50,000	S
7	52,000	S
8	57,000	S
9	62,410	← F
10	68,000	S
11	79,500	← F

F = Failed
S = Unfailed

Here are 11 vehicles in total , with four mileage intervals between failures , i. e.,

0	to	30,000 Miles	(interval #1)
30,000	to	46,005 Miles	(interval #2)
46,005	to	62,410 Miles	(interval #3)
62,410	to	79,500 Miles	(interval #4)

COMPUTATION OF THE NUMBER ACTIVE IN EACH MILEAGE INTERVAL

In mileage interval #1 there are

10 + 20,000/30,000 = 10.66667 active vehicles

In mileage interval #2 there are

7 + 5200/16,005 + 9010/16,005 = 7.88785 active vehicles

In mileage interval #3 there are

3 + 3995/16,405 + 5995/16,405 + 10,995/16,405

= 4.27918 active vehicles

In mileage interval #4 there are

1 + 5590/17,090 = 1.32709 active vehicles

SUMMARY OF FAILURES AND ACTIVE VEHICLES

Mileage Reached (at end of interval)	No. Failed (in interval)	No. Active (in interval)
30,000	1	10.66667
46,005	1	7.88785
62,410	1	4.27918
79,500	1	1.32709

MODIFICATION OF THE FAILURE RATES (I. E., ENTROPY INCREMENTS) IN EACH MILEAGE INTERVAL SO AS TO MAKE THEM APPROXIMATELY EQUAL TO MEDIAN FAILURE RATES (I. E., MEDIAN ENTROPY INCREMENTS)

For Mileage Interval #1 : (Both Numerator and Denominator Modified)

Use the fraction (No. Failed - .3/No. Active + .4)

$$\frac{1 - .3}{10.66667 + .4} = \frac{.7}{11.06667} = .06325 = \text{Median Entropy Increment}$$

For all subsequent intervals (after Interval #1) :

Use the fraction (No. Failed/No. Active + .4)

NOTE:
(the Numerator is not modified, but only the Denominator)

Thus ,

For Interval #2 : Median Entropy Increment = 1/7.88785 + .4
= 1/8.28785 = .12066

For Interval #3 : Median Entropy Increment = 1/4.27918 + .4
= 1/4.67918 = .21371

For Interval #4 : Median Entropy Increment = 1/1.32709 + .4
= 1/1.72709 = .57901

Thus , for the Entropy Plot with 50% confidence (corresponding to Median ranks) we have the following Table :

Mileage	Median Entropy Increment	Median Cumulative Entropy
30,000 miles	.06325	.06325
46,005 miles	.12066	.18391
62,410 miles	.21371	.39762
79,500 miles	.57901	.97663

From this Table we construct the Entropy paper plot shown in Figure 1 , which has Mileage as abscissa and Median Cumulative Entropy as ordinate . Both the abscissa and ordinate scales are logarithmic , so Entropy paper is simply Log-log paper with the Label Mileage horizontally and the Label Entropy vertically .

CONCLUSIONS FROM FIGURE 1

The Entropy Plot in Figure 1 tells us the following :

Weibull Slope = 2.75 (Slope of Entropy plot)

Characterisitic Life = 84,000 Miles (at Entropy = 1)

Median Life = 73,000 Miles (at Entropy = $\ln \frac{1}{.5}$) = ln 2 = .69315

B_{10} Life = 37,000 Miles (at Entropy = $\ln \frac{1}{.9}$) = .10536

Thus , by means of this example , we have shown the proper modifications which will convert an Entropy plot into Median parameters analogous to the employment of Median Ranks in elementary Weibull analysis . It should be emphasized that these modifications are not absolutely accurate , but are merely "Quick and dirty" empirical corrections .

ENTROPY PLOT OF THE VEHICLE DATA

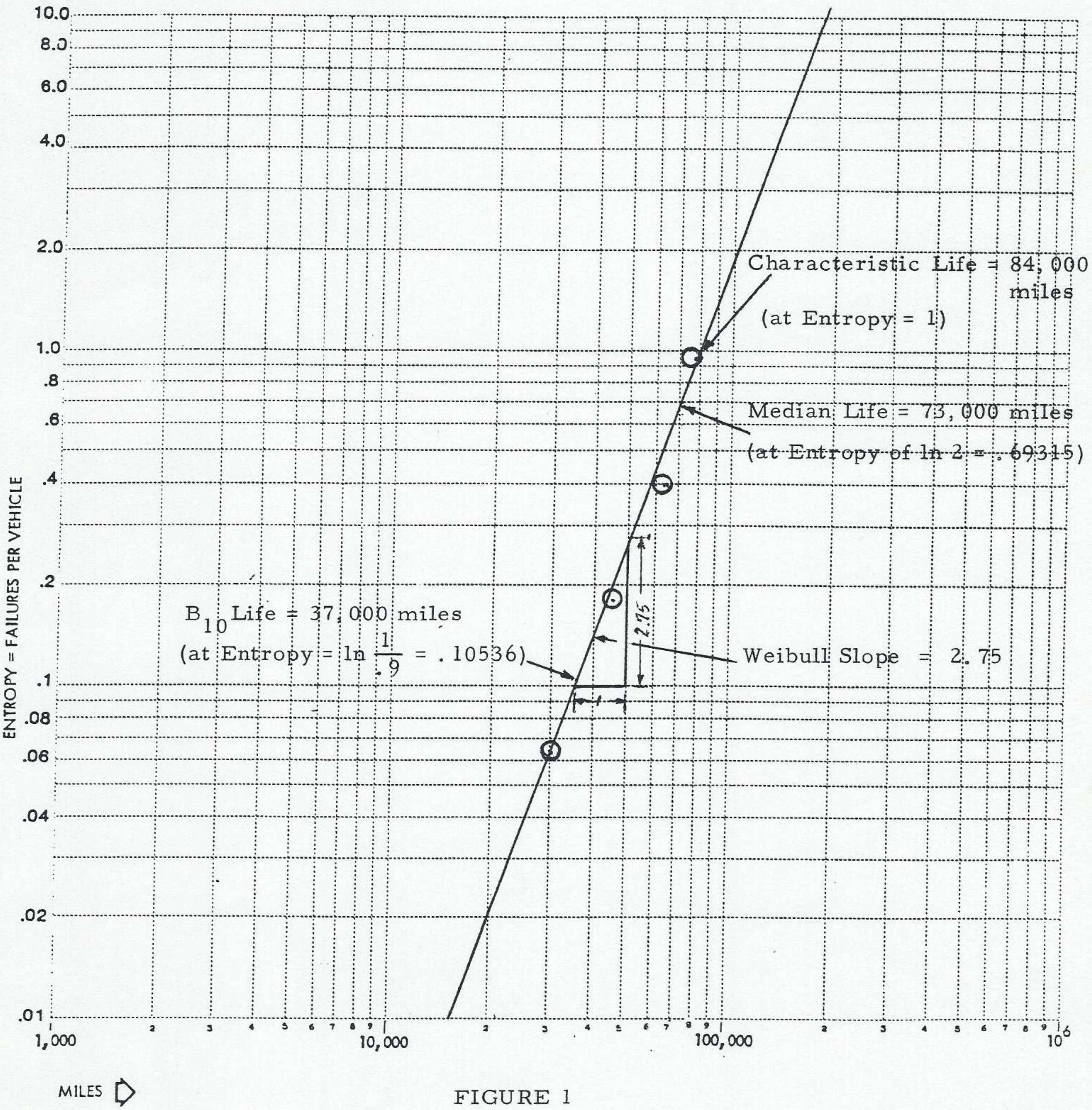

FIGURE 1