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THE LOG-PARAMETRIC APPROACH TO HANDLING TYPICAL SITUATIONS IN THE ECONOMICS OF TESTING PROGRAMS

INTRODUCTION

In a previous bulletin (Volume 14; Bulletin 5) , dated October 1984 , we discussed the Principles of Quantitative Reliability Management. The basic idea was that dollar gains and dollar losses determine the magnitude of a test and that the confidence involved is related to the C-Rank Theorem , which states that

$$R_c = \text{Reliability (with Confidence } c)$$

$$= 1 - \text{C-Rank of } (D+1)^{th} \text{ order statistics in } (N+1),$$

where

N = Number of items tested (to a target)

D = Number of defectives (out of the N)

(The defectives fail before target life is reached)

In this bulletin we want to employ <u>Log-Parametric C-Ranks</u> instead of Non-Parametric C-Ranks , because in most practical situations this makes sense. So , as a result , we can state

$$R_c = 1 - \text{Log-Parametric C-Rank of } (D+1)^{th} \text{ order statistics in } (N+1)$$

$$= \left(\frac{N - D + .7}{N + 1.4}\right)^{\left[(c/1-c)^{.55/\sqrt{N}}\right]}$$

With this as a basic relationship we present two typical situations and their solutions .

BASIC DEFINITIONS AND FORMULAS

SYMBOLS

G	=	Dollars Gained per Good Item
L	=	Dollars Lost per Bad Item
c	=	Confidence Level (in favor of reliability)
θ_c	=	Odds in Favor of Reliability when the Confidence is c
K_c	=	Profitability Factor with Confidence c .
R_c	=	Reliability with Confidence c .
N	=	Number of Items Tested (to a life target)
D	=	Number of Defectives (out of the N)

FORMULAS

$$R_c = \frac{K_c L}{K_c L + G}$$

$$\theta_c = \left[\frac{\ln\left(\frac{K_c L}{K_c L + G}\right)}{\ln\left(\frac{N - D + .7}{N + 1.4}\right)} \right]^{\frac{\sqrt{N}}{.55}}$$

$$c = \frac{\theta_c}{1 + \theta_c}$$

FIRST EXAMPLE OF A TYPICAL SITUATION

Suppose a test is run on 30 items to a desired target life of 1 000 hours with the following results :

28 items succeeded (ran 1000 hours without failure)

2 defective items (failed prior to 1000 hours)

QUESTION : If a good item (i. e., one which is able to survive 1000 hours) nets a gain of $100 , and if a defective item (i. e., one which does not last 1000 hours) causes a loss of $800 due to warranty costs and other inconveniences, how confident are we of at least breaking even on the product in the long run ? What is the Median Profitability Factor ?

SOLUTION

$$G = 100 , \quad L = 800 , \quad N = 30 , \quad D = 2 , \quad K_c = 1 .$$

$$\theta_c = \left[\frac{\ln\left(\frac{800}{800 + 100}\right)}{\ln\left(\frac{28.7}{31.4}\right)} \right]^{\frac{\sqrt{30}}{.55}} = \left(\frac{\ln .888889}{\ln .914013} \right)^{9.958592} = 14.718$$

$$c = \frac{\theta_c}{1 + \theta_c} = \frac{14.718}{15.718} = .9364 \qquad \text{(answer)}$$

Thus , there is 93.64% confidence of at least breaking even . The Median Profitability Factor (50% confidence) is given by the formula

$$K_{.50} = \frac{(N - D + .7)\, G}{(D + .7)\, L} = \frac{(28.7)(100)}{(2.7)(800)} = \frac{2870}{2160} = 1.329 \qquad \text{(answer)}$$

(50% confident that gains will be at least 1.329 times as large as loses)

SECOND EXAMPLE OF A TYPICAL SITUATION

It is known that each bad item (which does not last the required life) costs us \$250 , while each good item (which lasts the required life) gain us \$100 .

How large a sample N must be tested to the required life with 3 bad in order to give us 99% confidence of gaining twice as much as we lose in the long run ?

SOLUTION

$c = .99 , \quad D = 3 , \quad K_c = 2 , \quad L = 250 , \quad G = 100 .$

$$\theta_c = \frac{c}{1 - c} = \frac{.99}{1 - .99} = \frac{.99}{.01} = 99$$

Try N = 40

$$\theta_c = \left[\frac{\ln\left(\frac{500}{500 + 100}\right)}{\ln\left(\frac{37.7}{41.4}\right)} \right]^{\frac{\sqrt{40}}{.55}} = \left(\frac{\ln .833333}{\ln .910628} \right)^{11.4199191} = 2131.24$$

$$c = \frac{2131.24}{2132.24} = .9995 \qquad \text{(too many tested)}$$

Try N = 25

$$\theta_c = \left[\frac{\ln\left(\frac{500}{500+100}\right)}{\ln\left(\frac{22.7}{26.4}\right)} \right]^{\frac{\sqrt{25}}{.55}} = \left(\frac{\ln .833333}{\ln .859846} \right)^{9.090909} = 5.5489$$

$$c = \frac{5.5489}{6.5489} = .8473 \qquad \text{(not enough tested)}$$

Try N = 30

$$\sigma_c = \left[\frac{\ln\left(\frac{500}{500 + 100}\right)}{\ln\left(\frac{27.7}{31.4}\right)}\right]^{\frac{\sqrt{30}}{.55}} = \left(\frac{\ln .833333}{\ln .882165}\right)^{9.9958592} = 41.64$$

$$c = \frac{41.64}{42.64} = .9765 \quad \text{(not enough tested)}$$

Try N = 32

$$\sigma_c = \left[\frac{\ln\left(\frac{500}{500 + 100}\right)}{\ln\left(\frac{29.7}{33.4}\right)}\right]^{\frac{\sqrt{32}}{.55}} = \left(\frac{\ln .833333}{\ln .889222}\right)^{10.28519} = 92.45$$

$$c = \frac{92.45}{93.45} = .9893 \quad \text{(not quite enough tested)}$$

Try N = 33

$$\sigma_c = \left[\frac{\ln\left(\frac{500}{500 + 100}\right)}{\ln\left(\frac{30.7}{34.4}\right)}\right]^{\frac{\sqrt{33}}{.55}} = \left(\frac{\ln .833333}{\ln .892442}\right)^{10.44466} = 137.48$$

$$c = \frac{137.48}{138.48} = .9928 \quad \text{(slightly over the desired 99\%)}$$

Thus , we conclude that we must test 33 items to the required life and if 30 out of the 33 make it , we have 99.28% confidence of gaining twice as much money as we lose .

CONCLUSION

It can be seen that the Log-Parametric Ranking approach is a powerful tool in resolving sample size questions , as well as profitability questions arising out of testing programs for reliabilities of products required to last for specific life targets .