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A COMPUTER PROGRAM FOR DETERMINING A CONFIDENCE INTERPOLATION DIAGRAM FOR PROFIT FACTORS AS PREDICTED FROM RELIABILITY TESTS

INTRODUCTION

The first rule for an adequate reliability testing program can be stated as follows :

RULE # 1 : In order to determine the required reliability we must know Failure Costs and Success Profits .

In addition to the failure costs and success profits of Rule # 1 , we must also specify the Desired Profitability Factor K , such that the ratio

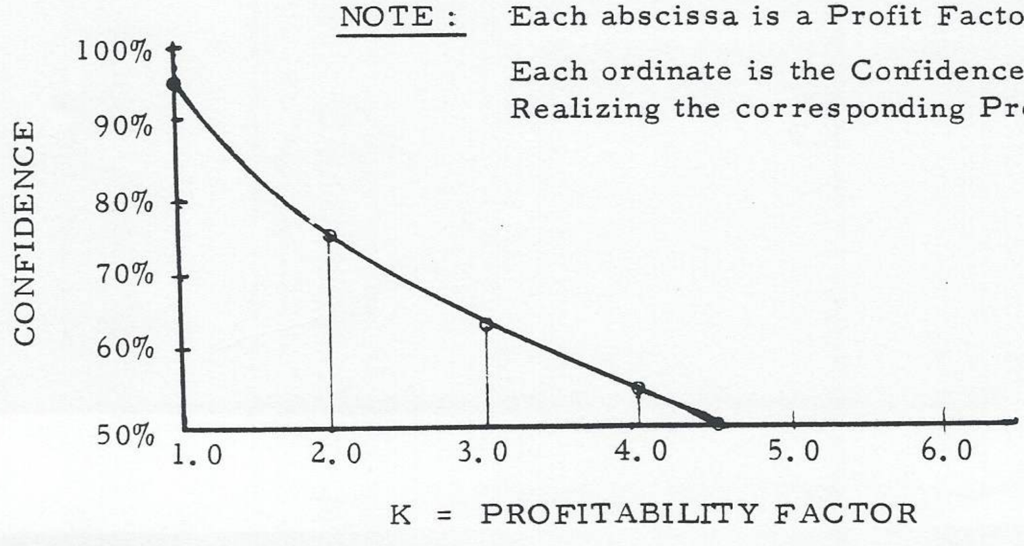
$$\frac{\text{Gains from Successes}}{\text{Losses from Failures}} = K \quad .$$

This simply means that we want to have enough reliability to a specified service length to enable us to gain K times as much money from Good Items (which survive the specified service length) as we lose from Bad Items (which fail to last the specified service length) .

A Confidence Interpolation Diagram of Profitability Factors would look as follows

NOTE : Each abscissa is a Profit Factor K .

Each ordinate is the Confidence of Realizing the corresponding Profit Factor.



In this bulletin we shall list a computer program in BASIC Language which calculates the Profitability Factor K corresponding to any desired confidence, having given

G = Dollars gained per good item

L = Dollars lost per bad item

D = No. of bad items in a test of N items to target

$R_{min.}$ = Minimum Reliability (at which we break even) = $\frac{L}{L+G}$

THE BASIC SAMPLE SIZE FORMULA

From elementary Reliability Theory we can state that if D defectives are observed in N trials, then the Median Reliability (by using Benard's approximation) is

$$R_{.50} = \frac{N - D + .7}{N + 1.4} \qquad (1)$$

If S = No. of Successes = $N - D$, then this can also be written as

$$R_{.50} = \frac{S + .7}{N + 1.4} \qquad (2)$$

We can then determine the Median Profitability Factor $K_{.50}$ (with 50% confidence) as follows

Let G = Dollars gained per reliable item

Let L = Dollars lost per failed item

Then

$$K_{.50} = \frac{G R_{.50}}{L(1 - R_{.50})} = \frac{G\left(\frac{N - D + .7}{N + 1.4}\right)}{L\left(1 - \frac{N - D + .7}{N + 1.4}\right)}$$

or

$$K_{.50} = \frac{G(N - D + .7)}{L(D + .7)} \qquad (3)$$

Thus, if we specify the Median Profitability Factor Desired (i.e., $K_{.50}$) from a test with D defectives, we can determine the required sample size N from (3) by solving for N, thus,

$$\boxed{N = \frac{L K_{.50}(D + .7)}{G} + (D - .7)} \qquad (4)$$

(4) is the basic sample size formula ..

THE PROFITABILITY FACTOR FOR ANY DESIRED CONFIDENCE

Let K_c = Profitability Factor with Confidence c .

Then

$$K_c = \frac{G R_c}{L(1 - R_c)} \tag{5}$$

Where R_c = Reliability with Confidence c from a test with D defectives in N items .

According to the Log-Parametric Theory :

R_c = 1 - log-parametric c-rank of $(D + 1)^{\underline{th}}$ order statistics

$$\text{in } (N + 1) = \left(\frac{N - D + .7}{N + 1.4}\right)^{\left[(c/1 - c)^{.55/\sqrt{N}}\right]} \tag{6}$$

After determining R_c from (6), we substitute it into (5) to obtain

$$K_c = \frac{G\left(\frac{N - D + .7}{N + 1.4}\right)^{\left[(c/1 - c)^{.55/\sqrt{N}}\right]}}{L\left\{1 - \left(\frac{N - D + .7}{N + 1.4}\right)^{\left[(c/1 - c)^{.55/\sqrt{N}}\right]}\right\}}$$

$$K_c = \frac{G(N - D + .7)^{\mu}}{L\left[(N + 1.4)^{\mu} - (N - D + .7)^{\mu}\right]} \qquad \text{define } \mu = \left(\frac{c}{1 - c}\right)^{\frac{.55}{\sqrt{N}}}$$

$$K_c = \frac{G}{L\left[\left(\frac{N + 1.4}{N - D + .7}\right)^{\mu} - 1\right]}$$

$$K_c = \frac{G}{L\left[\left(1 + \frac{G}{K_{.50} L}\right)^{\mu} - 1\right]} \tag{7}$$

Thus, Formula (7) gives us the Profitability Factor corresponding to any desired confidence c , where

$$\mu = (c/1 - c)^{.55/\sqrt{N}} \qquad (N = \text{sample size})$$

THE COMPUTER PROGRAM IN BASIC LANGUAGE

Suppose $G = \$100$ (Gain per Good Item)

$L = \$250$ (Loss per Bad Item)

$D = 3$ defectives in N trials

$R_{min.} = L/L + G = 250/350 = .714286$

$K_{.50}$ = Median Profitability Factor Desired = 4

With these inputs we write the following program in BASIC language :

```
160   DATA 3, 100, 250, 4
170   READ D, G, L, K
180   PRINT "GAIN PER GOOD ITEM=" ; G
190   PRINT "LOSS PER BAD ITEM ="; L
200   PRINT "NO. DEFECTIVE =" ; D
210   PRINT "MEDIAN PROFIT FACTOR DESIRED =" ; K
220   R1 = L/(L + G)
230   PRINT "MIN. REL.=" ; R1
240   N = (K * L * (D + .7))/G + D - .7
250   PRINT "SAMPLE SIZE REQUIRED = " ; N
260   H1 = LOG (L/(L + G))
270   H2 = LOG ((N - D + .7)/(N + 1.4))
280   H3 = H2/H1
290   H4 = H3 ^ ((N ^ .5)/.55)
300   C1 = 1/(1 + H4)
310   PRINT "CONF. OF R MIN. = " ; C1
320   C = .9  <----(NOTE: Insert as many different values as needed
                     for the Confidence Interpolation Diagram)
330   PRINT "CONF. DESIRED FOR PROFIT FACTOR = " ; C
340   E1 = C/(1 - C)
350   E2 = .55/(N ^ .5)
360   E3 = E1 ^ E2
370   E4 = (1 + G/(K * L)) ^ E3
380   E5 = L * (E4 - 1)
390   K1 = G/E5
400   PRINT "PROFIT FACTOR WITH DESIRED CONF.=" ; K1
410   END
```

THE PROGRAM PRINTOUT

When the program with the indicated inputs is run we get a printout as follows :

GAIN PER GOOD ITEM = 100

LOSS PER BAD ITEM = 250

NO. DEFECTIVE = 3

MEDIAN PROFIT FACTOR DESIRED = 4

MIN. REL. = .714286

SAMPLE SIZE REQUIRED = 39.3

CONF. OF R MIN. = .99999943

CONF. DESIRED FOR PROFIT FACTOR = .90

PROFIT FACTOR WITH DESIRED CONF. = 3.265

CONCLUSION

It can be seen how handily this computer program determines the profit factors for different confidence levels in a given testing program.

In the example given, where G = 100 , L = 250, D = 2 , and $K_{.50} = 4$. we calculate the following Table from which we can construct the graph of the Confidence Interpolation Diagram for Profit Factors :

CONFIDENCE	PROFIT FACTOR
.50	4.000
.60	3.853
.70	3.700
.80	3.520
.90	3.265
.95	3.045
.99	2.609
.999	2.095

This is truly a useful program in the economics of reliability test planning .