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DETROIT RESEARCH INSTITUTE
P.O. BOX 36504 • GROSSE POINTE, MICHIGAN 48236 • (313) 886-7976

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

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# "QUICK AND DIRTY" FORMULAS FOR CALCULATING THE ODDS THAT A PRODUCT LIFE TEST SHOWS SUPERIORITY TO A STANDARD OR PREVIOUS DESIGN

## INTRODUCTION

One of the everyday problems we faced with when evaluating a proposed design is the problem of demonstrating sufficient evidence from a life test that the design complies with a required standard (say , a $B_{10}$ life) , or , perhaps , that the design is at least as good as another design for which we have durability data. The purpose of this bulletin is to present some easy-to-use techniques for quickly calculating the odds that a design's life at any quantile level (say $B_{10}$) is at least as good as a standard required life at the same quantile level . Also , the same question of odds can be answered by a more general formula when comparing a present design life test to a previous life data set on another design , and thus determining the evidence in favor of the newer design (say , at $B_{10}$ life ) .

These practical approaches should be classified as "Quick and Dirty" , and thus the question of rigorous validity will not be discussed. Let it be said only that all of this author's experienced has validated these approaches from the fact that they have yet to turn out disappointing. Thus , by looking at the odds required to overcome product failure losses by profits from reliable items , we will know from these "Quick and Dirty" formulas whether or not our test data permit product acceptance .

## COMPARING A DESIGN'S WEIBULL PLOT TO A STANDARD $B_{10}$

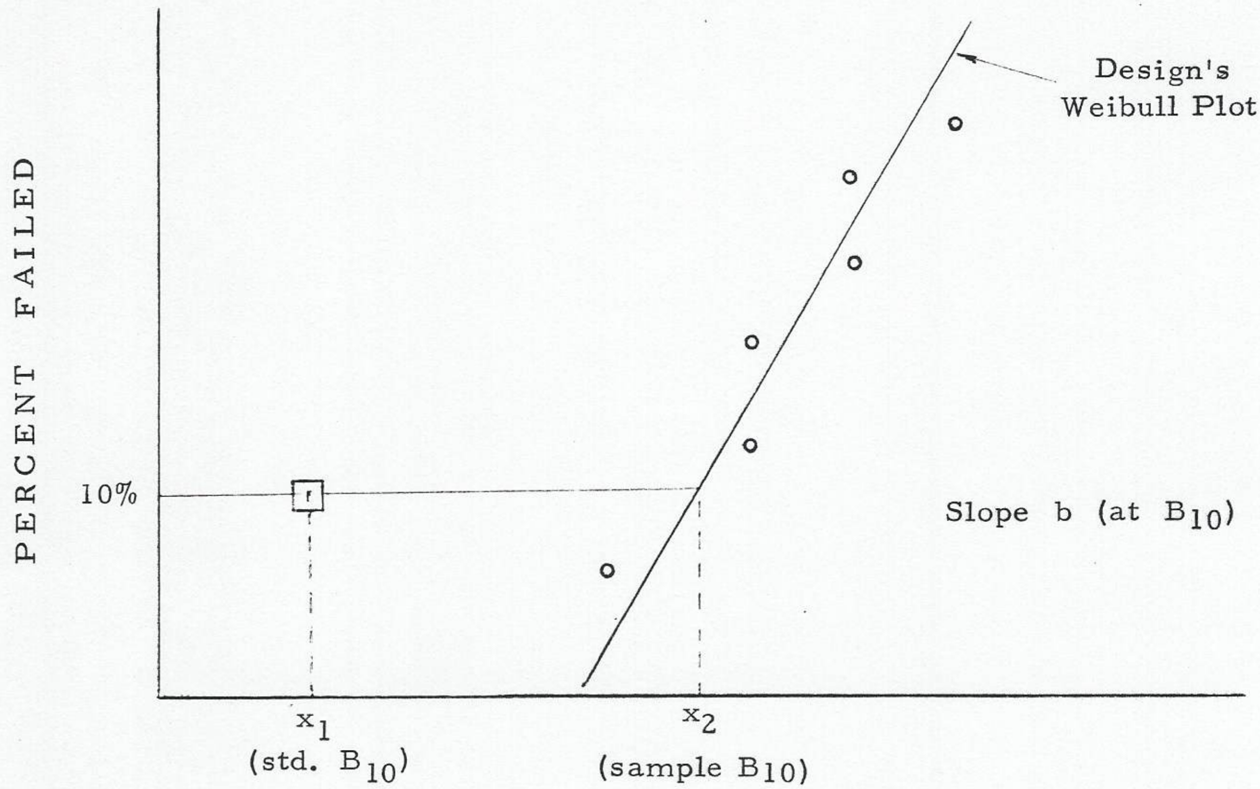


FIGURE 1

(Weibull Plot of N items of Design Tested to Failure)

DEFINE : LIFE RATIO $= \dfrac{\text{SAMPLE } B_{10}}{\text{STD. } B_{10}} = \dfrac{x_2}{x_1}$ (1)

DEFINE : ODDS EXPONENT = (DESIGN's Weibull Slope)$\sqrt{\text{Sample size}}\,/(\sqrt{3}/\pi)$

Since $\sqrt{3}/\pi$ = .55 (approx.), and b = Design's Weibull Slope (at $B_{10}$)
and Sample size = N , it follows that

$$\text{ODDS EXPONENT} = \frac{b\sqrt{N}}{.55} \qquad (2)$$

Then , we define

$$\text{ODDS} = (\text{LIFE RATIO})^{(\text{ODDS EXPONENT})} \qquad (3)$$

or

$$\text{ODDS} = (x_2/x_1)^{(b\sqrt{N}/.55)} \qquad (4)$$

## EXAMPLE OF AN ODDS CALCULATION (FOR A DESIGN SAMPLE vs. A STANDARD $B_{10}$)

Suppose the $B_{10}$ life is required to be at least 500 hours in service . Suppose a design is tested under service conditions by using a sample of 6 with the following results :

| j<br>Failure No. | x<br>Hours to Failure | $\frac{j-.3}{N+.4}$<br>Median Rank |
|---|---|---|
| 1 | 846 hrs. | .1094 |
| 2 | 1519 hrs. | .2656 |
| 3 | 2358 hrs. | .4219 |
| 4 | 3440 hrs. | .6781 |
| 5 | 4610 hrs. | .7344 |
| 6 | 5951 hrs. | .8906 |

We construct the Weibull plot for this data as shown in Figure 2 below :

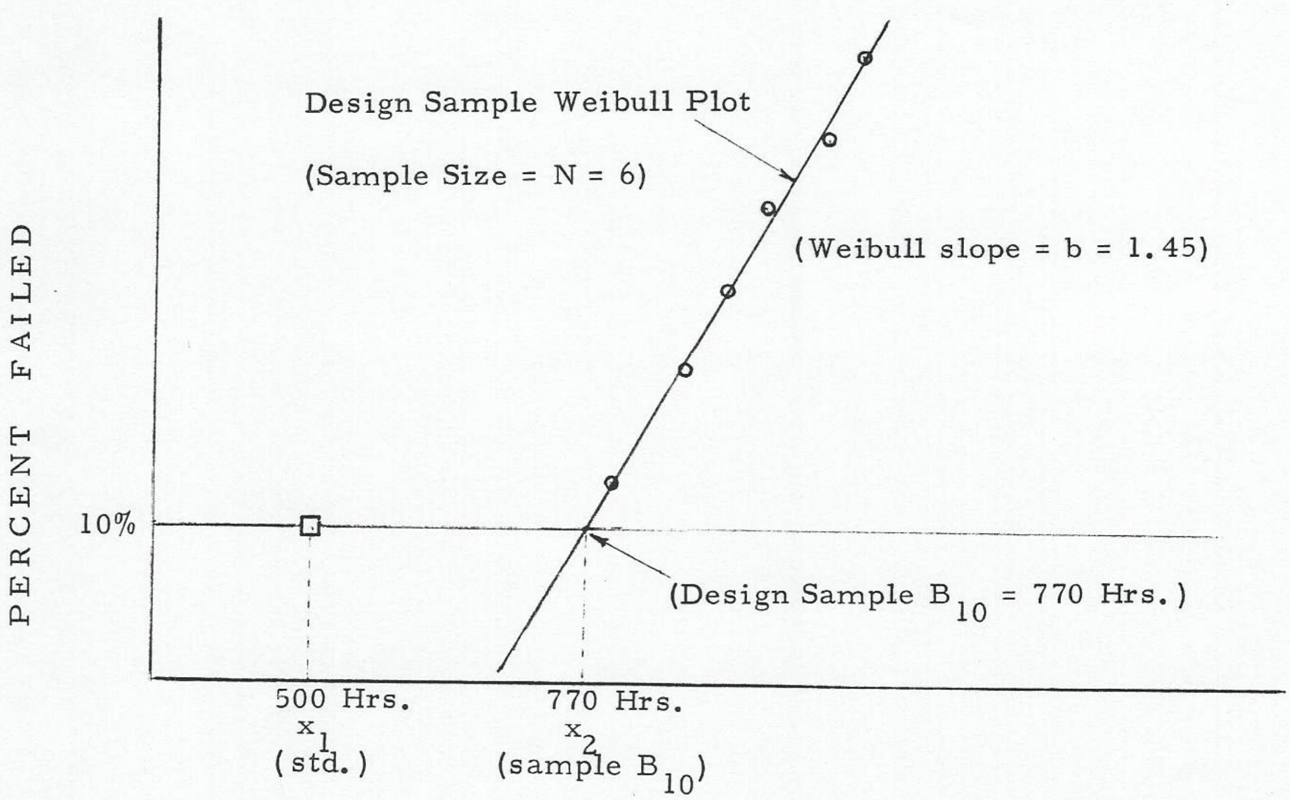


FIGURE 2

LIFE RATIO = $x_2/x_1$ = 770/500 = 1.54

ODDS EXPONENT = $b\sqrt{N}/.55$ = $1.45\sqrt{6}/.55$ = 6.45

ODDS = $(1.54)^{6.46}$ = 16.3 (ans.)

Thus, the Odds in favor of being at least as good as the goal of $B_{10}$ at 500 hours are better than 16 : 1. This means that as long as the money ratio

$$\left(\frac{\text{Losses from Not Meeting the Goal}}{\text{Gains from Meeting the Goal}}\right)$$

is less than 16, the producer shall be able to realize a profit.

## COMPARING TWO DIFFERENT DESIGNS (BY LOOKING AT THEIR WEIBULL PLOTS)

A more general situation presents itself when we have separate Weibull plots for two different designs. This is depicted in Figure 3, in which Design I has

$$\begin{pmatrix} b_1 = \text{Weibull slope of I} \\ x_1 = B_{10} \text{ Life of I} \\ N_1 = \text{Sample Size of I} \end{pmatrix}$$

and Design II has

$$\begin{pmatrix} b_2 = \text{Weibull slope of II} \\ x_2 = B_{10} \text{ Life of II} \\ N_2 = \text{Sample Size of II} \end{pmatrix}$$

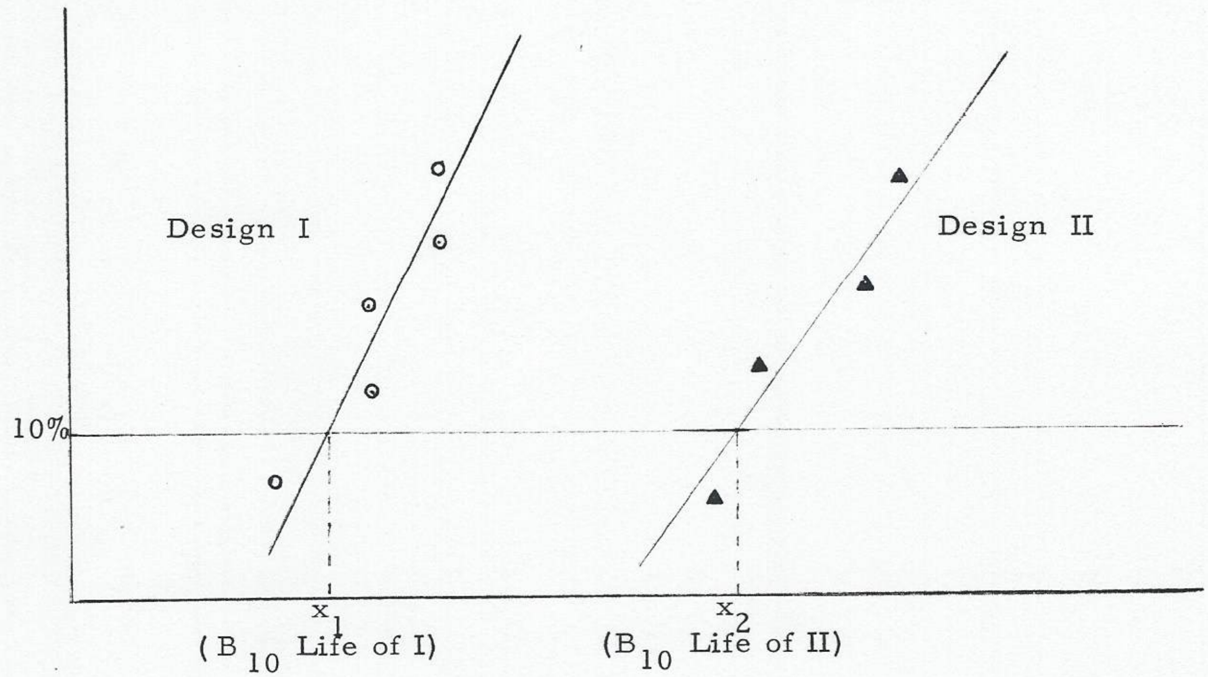


FIGURE 3

QUESTION : What are the Odds that the $B_{10}$ life of Design II is better than the $B_{10}$ life of Design I ?

A "Quick and Dirty" formulas for the desired odds (that Design II is better than Design I at $B_{10}$) is

$$\text{ODDS} = (\text{LIFE RATIO})^{(\text{ODDS EXPONENT})} \tag{5}$$

Where

$$\text{LIFE RATIO} = \frac{x_2}{x_1} \tag{6}$$

NOTE: $x_1$ and $x_2$ are both at the same quantile level. In the illustration (Figure 3) both $x_1$ and $x_2$ are at 10% failed.

$$\text{ODDS EXPONENT} = \frac{\sqrt{1 + \dfrac{\sqrt{N_1 N_2}}{1/2(N_1 + N_2)}}}{.55\left(\dfrac{1}{b_1\sqrt{N_1}} + \dfrac{1}{b_2\sqrt{N_2}}\right)}, \tag{7}$$

where $b_1$ and $b_2$ are Weibull slope of I and II, respectively, and $N_1$ and $N_2$ are the sample size of I and II respectively. These slopes and sample sizes are taken at the quantile level where $x_1$ and $x_2$ are located in Figure 3.

NOTE: In case $N_1 = \infty$, the odds exponent in (7) becomes $b_2\sqrt{N_2}/.55$, which is the same as formula (4) when the left hand value is made into a fixed standard $x_1$.

## NUMERICAL EXAMPLE OF COMPARING TWO DESIGNS

Let us say we have 4 specimens of Design I and obtain the left Weibull plot in Figure 4 . Also we test 6 specimens of Design II and obtain the right Weibull plot in Figure 4 .

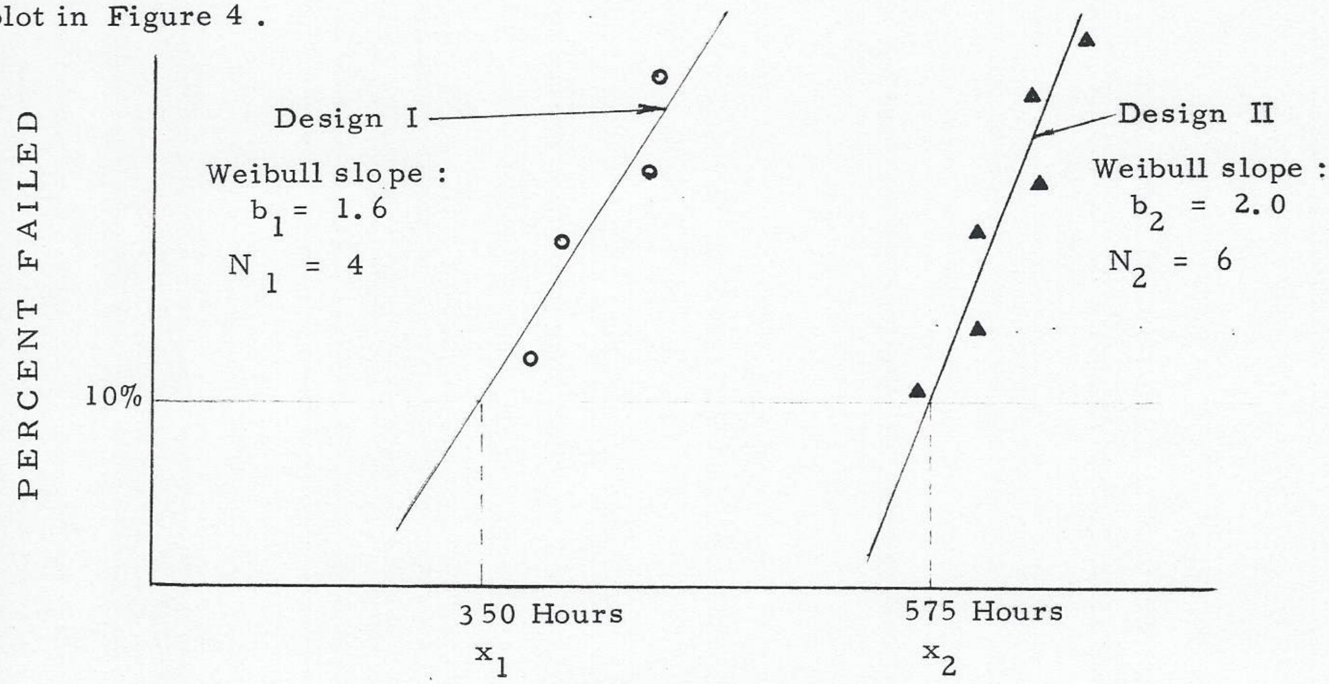


FIGURE 4

In this case , LIFE RATIO $= \dfrac{x_2}{x_1} = \dfrac{575}{350} = 1.64$

$$\text{ODDS EXPONENT} = \frac{\sqrt{1 + \frac{\sqrt{24}}{5}}}{.55\left(\frac{1}{1.6\sqrt{4}} + \frac{1}{2\sqrt{6}}\right)} = \frac{1.40705}{.28414} = 4.95$$

$$\therefore \text{ODDS} = (1.64)^{4.95} = 11.57 \qquad \text{(ans.)}$$

In this situation , the calculated Odds of 11.57 to 1 would be sufficient for profitability in case

$$\left(\frac{\text{LOSSES WHEN I IS BETTER}}{\text{GAINS WHEN II IS BETTER}}\right) < 11.57$$

(Assuming we had bet on II as being really better)

## CONCLUSIONS

It can be seen that the convenient general formula

$$\text{ODDS} = (\text{LIFE RATIO})^{(\text{ODDS EXPONENT})}$$

is truly a handy and quick formula for estimating Odds in favor of a Superior product with respect to either

(a) a fixed standard

(b) a previous Design's Weibull plot .

As was stated in the introduction , we have not discussed the rigorous aspects of these " Quick and Dirty " techniques , but we want to reiterate that this approach has never disappointed us over the past 40 years of experience in making such product decisions .