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FORMULAS FOR THE EVIDENCE OF MEETING VARIOUS RELIABILITY GOALS

INTRODUCTION

In dealing with statistical situations in life testing we are always faced with the fact that no conclusion involving any hypothesis is ever absolutely certain to be correct. What we must do is gather enough evidence in favor of the hypothesis. For example, one test alone may not yield sufficient evidence of life improvement, but additional tests which point in the same direction will allow us to conclude that there is improvement due to the fact that the evidence is always in the same direction (i.e., always positive). This produces a build-up of accumulated evidence by allowing us to add on each new amount of positive evidence on top of any previous evidence.

The mathematical definition of Evidence in Favor of any hypothesis is

$$\text{EVIDENCE} = \ln(\text{ODDS IN FAVOR})$$

Since

$$\text{ODDS IN FAVOR} = \frac{\text{CONFIDENCE IN FAVOR}}{1 - \text{CONFIDENCE IN FAVOR}}$$

We can write

$$\text{CONFIDENCE IN FAVOR} = \frac{1}{1 + e^{-\text{EVIDENCE}}}$$

Thus , Confidence depends on Evidence , and Evidence for various situations is the topic of discussion in this bulletin .

SITUATION I : EVIDENCE OF MEETING A B_Q LIFE GOAL

Suppose the desired B_q life has been specified as being x_o hours. Some test data with sample size N at B_q life yield a test value

$$B_q \text{ Life} = x_{test} \text{ Hours}.$$

STATISTICAL ASSUMPTION

B_q test life has a log-normal distribution with a median of $\ln x_{test}$ and a median standard deviation of $1/b_{test}\sqrt{N(.5 + .5q)}$

where b_{test} = Weibull slope of the test data plot at B_q

CALCULATION OF Z-SCORE TO x_o HOURS

The formula for the Z-score in a log-normal distribution with a mean value at $\ln x_{test}$ and a median standard deviation of $b_{test}\sqrt{N(.5 + .5q)}$ is

$$Z = \frac{\ln x_o - \ln x_{test}}{\dfrac{1}{b_{test}\sqrt{N(.5 + .5q)}}} = -b_{test}\sqrt{N(.5 + .5q)}\,\ln\left(\frac{x_{test}}{x_o}\right)$$

The formula for EVIDENCE is

$$\text{EVIDENCE} = \frac{\pi}{\sqrt{3}}|Z| = \frac{\pi}{\sqrt{3}}\,b_{test}\sqrt{N(.5 + .5q)}\,\ln\left(\frac{x_{test}}{x_o}\right)$$

The formula for CONFIDENCE is $\dfrac{1}{1 + e^{-\text{EVIDENCE}}}$

SITUATION II : MEETING A GOAL REPRESENTED BY A PREVIOUS DESIGN'S LIFE TEST SAMPLE

Suppose that a previous design was tested for durability with a sample of N_o items, and that a B_q life on Weibull paper was x_o hours with a two-parameter Weibull line fitted by the parameters

$$\left(\begin{array}{lcl} \theta_o & = & \text{Sample Characteristic Life} \\ b_o & = & \text{Sample Weibull Slope} \end{array} \right)$$

Next, suppose a new design is tested by using a sample of N_1 items tested to failure, yielding a B_q life of x_1 hours, together with Weibull parameters

$$\left(\begin{array}{lcl} \theta_1 & = & \text{New Deign's Characteristic Life} \\ b_1 & = & \text{New Design's Weibull Slope} \end{array} \right)$$

QUESTION: What is the Median Evidence that the New Design is superior to the previous design at B_q life ?

ANSWER :

The EVIDENCE we seek is given by the formula

$$\text{EVIDENCE} = \frac{\frac{\pi}{\sqrt{3}} K \ln\left(\frac{x_1}{x_o}\right)}{\frac{1}{b_o\sqrt{N_o(.5 + .5q)}} + \frac{1}{b_1\sqrt{N_1(.5 + .5q)}}}$$

Where $K = \sqrt{1 + \frac{\sqrt{N_o N_1}}{\frac{1}{2}(N_o + N_1)}}$

THE FORMULA FOR CONFIDENCE

$$\text{CONFIDENCE} = \frac{1}{1 + e^{-\text{EVIDENCE}}}$$

EXAMPLES

EXAMPLE #1 : A B_{10} life goal is 1000 hours. A sample of 5 items is tested (all to failure) with the followong results :

FAILURE NO.	HOURS	MEDIAN RANK
1	1750	.1296
2	3100	.3148
3	5225	.5000
4	7790	.6852
5	10,600	.8704

Plotting these on Weibull paper yields Figure 1, which shows

$$\left[\begin{array}{l} \theta_{test} = \text{Test Characteristic Life} = 6644 \text{ Hours} \\ b_{test} = \text{Test Weibull Slope} = 1.44 \\ {}_{test}B_{10} \text{ Life} = 1385 \text{ Hours} \end{array}\right]$$

So, the Z-score we are interested in is

$$Z = b_{test}\sqrt{N(.5 + .5(.1))}\ \ln\left(\frac{1385}{1000}\right) \qquad \text{(NOTE: } q=.1 \text{ at } B_{10})$$

$$Z = 1.44\sqrt{5(.55)}\ \ln 1.385 = .7778$$

The EVIDENCE is then $\frac{\pi}{\sqrt{3}}(.7778) = 1.411$ Units.

Hence, the CONFIDENCE of meeting the B_{10} life goal of 1000 hours is

$$C = \frac{1}{1 + e^{-1.411}} = .80$$

DATA PLOT FOR EXAMPLE # 1

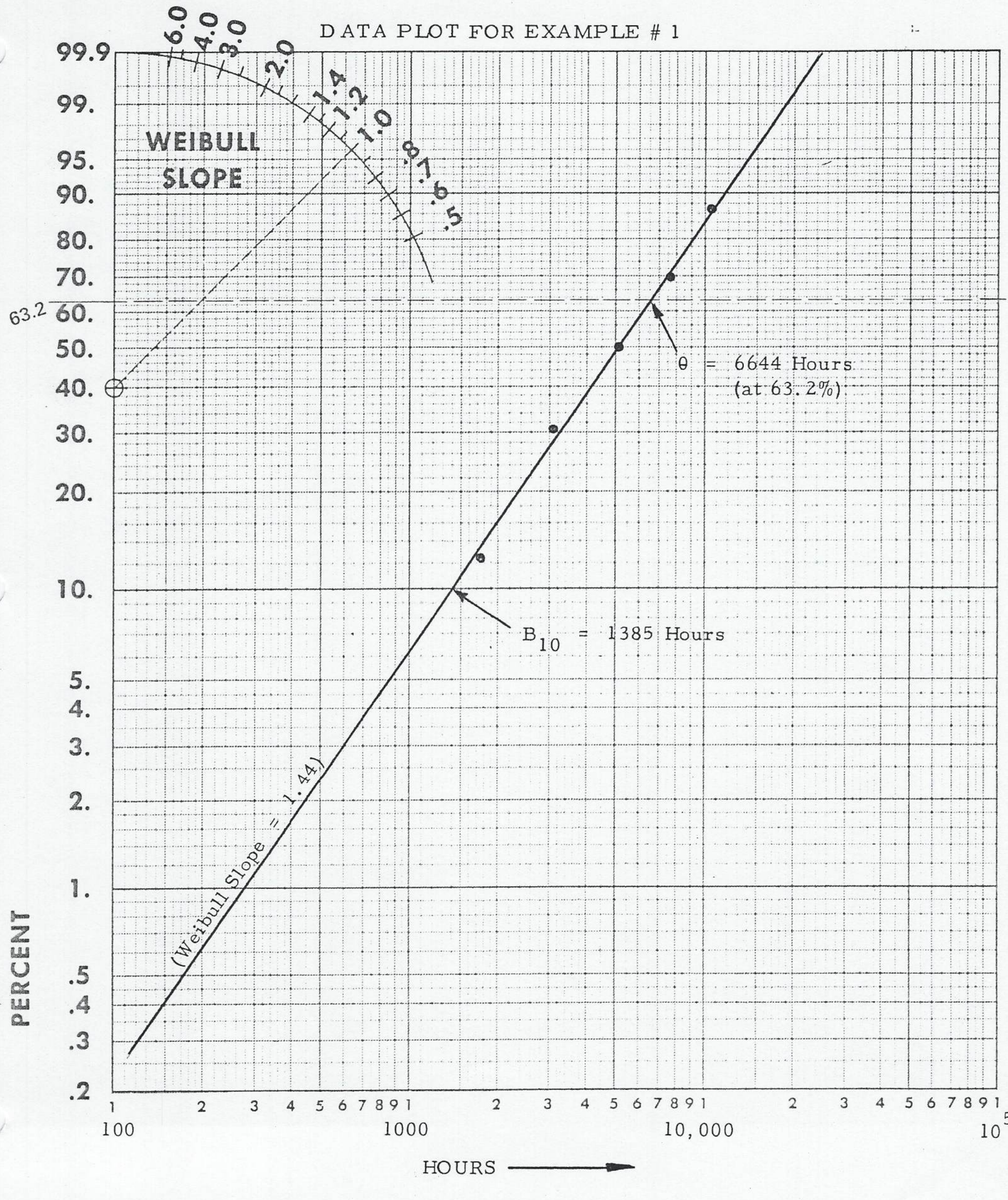

FIGURE 1

EXAMPLE #2: A previous design was tested by failing 5 specimens, and a Weibull plot gave

$$\begin{bmatrix} b_o = 2.1 \\ \theta_o = 1200 \text{ Hours} \\ {}_oB_{10} = 411 \text{ Hours} \end{bmatrix}$$

A new design is tested by failing 8 specimens , which yield

$$\begin{bmatrix} b_1 = 2.5 \\ \theta_1 = 1825 \text{ Hours} \\ {}_1B_{10} = 742 \text{ Hours} \end{bmatrix}$$

The EVIDENCE of improved B_{10} life is therefore

$$\text{EVIDENCE} = \frac{\frac{\pi}{\sqrt{3}}\sqrt{1 + \frac{\sqrt{40}}{6.5}}\ \ln\left(\frac{742}{411}\right)}{\frac{1}{2.1\sqrt{5(.5+.5(.1))}} + \frac{1}{2.5\sqrt{8(.5+.5(.1))}}}$$

(NOTE : $q = .1$ at B_{10} life)

$$\text{EVIDENCE} = \frac{1.8138\ (1.4046)\ \ln\ (1.805)}{.28715 + .19069} = \frac{1.50455}{.47784} = 3.149 \text{ Units}$$

Hence, The CONFIDENCE that the New Design has a better B_{10} life than the previous design is

$$C = \frac{1}{1 + e^{-3.149}} = .959$$

CONCLUSION

It can be seen that the EVIDENCE technique is a handy way of evaluating the CONFIDENCE that we have an improvement in durability, both in the case

(a) Where a B_q life requirement is specified and we want to know how well a tested design stacks up against such a requirement.

(b) Where a New Design is to be compared to a previous design at some quantile level (i.e., B_q life level).