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CONSTRUCTING NOMINAL 90% CONFIDENCE BANDS ON WEIBULL PAPER

INTRODUCTION

One of the basic rules in reliability anlaysis from failure data is " let everyone concerned see the confidence band for any prediction. "

This fundamental rule for statistical predictions can be implemented only if we know what is the theoretical basis for calculating confidence limits on any B_q life in an estimated cumulative distribution function of life .

In this bulletin we shall give the basic formula for the Nominal Standard Deviation of the logarithm of B_q life in terms of three factors :

(1) The Quantile Level q

(2) The Sample Size N at Quantile q *

(3) The Weibull Slope at Quantile q

From this Nominal Standard Deviation of the logarithm of B_q life we proceed to calculate the lower 5% and upper 95% limits on the B_q life by assuming that each B_q life has a log-normal distribution with the desired limits at $\pm$ 1.645 sigmas away from the central estimate of B_q life in a Weibull plot .

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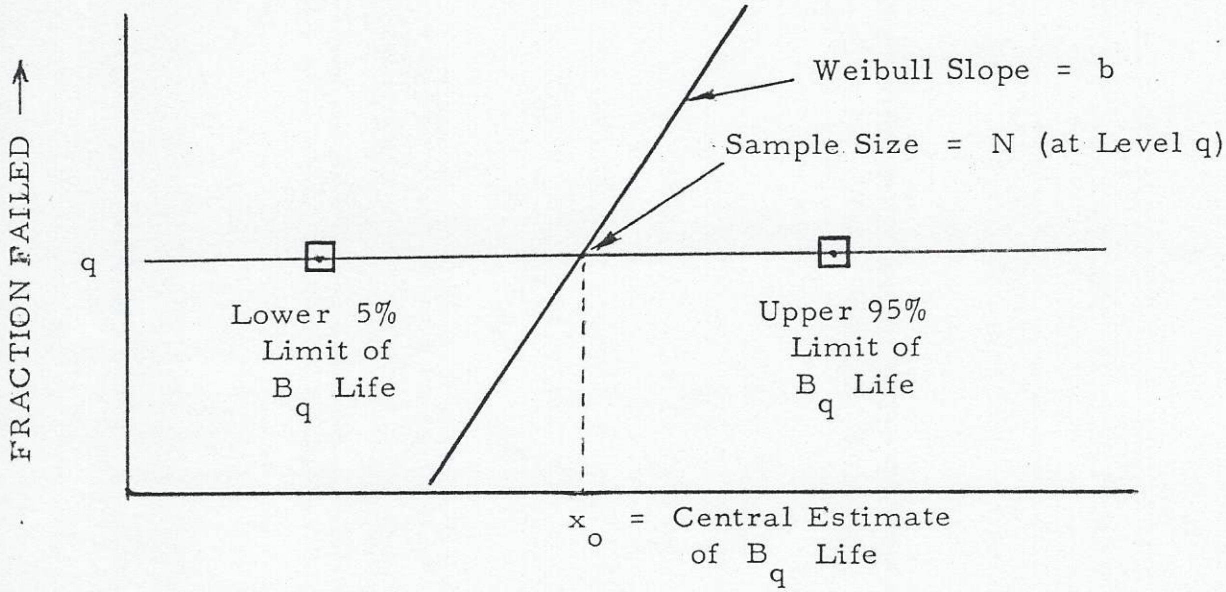
* N = Total in data set - No. Suspended prior to B_q life

THE BASIC SIGMA FORMULA FOR THE LOGARITHM OF B_Q LIFE

In Figure 1 below we have a line of slope b on Weibull paper which shows a life x_o at quantile level q . In other words , the estimated B_q life has a central predicted value of x_o

FIGURE 1



The formula for the MEDIAN STANDARD DEVIATION of the logarithm of the B_q life is

$$\sigma_{\ln B_q} = \frac{1}{b\sqrt{N(.5 + .5q)}} \qquad (q \leq .5)$$

If $q > .5$, then the formula is

$$\sigma_{\ln B_q} = \frac{1}{b\sqrt{N[.5 + .5(1 - q)]}}$$

FORMULA FOR THE LOWER 5% LIMIT

$$\text{MEAN} = \ln x_o$$

$$\text{SIGMA} = \frac{1}{b\sqrt{N(.5 + .5q)}} \qquad (q \leq .5)$$

$$\text{LOWER 5\% LIMIT of } \ln B_q = \text{MEAN} - 1.645 \text{ SIGMA}$$

$$= \ln x_o - \frac{1.645}{b\sqrt{N(.5 + .5q)}} \qquad (q \leq .5)$$

Therefore , the LOWER 5% LIMIT of B_q life is

$$x_o e^{-\frac{1.645}{b\sqrt{N(.5 + .5q)}}} \qquad (q \leq .5)$$

FORMULA FOR THE UPPER 95% LIMIT

$$\text{UPPER 95\% LIMIT of } \ln B_q = \text{MEAN} + 1.645 \text{ SIGMA}$$

$$= \ln x_o + \frac{1.645}{b\sqrt{N(.5 + .5q)}} \qquad (q \leq .5)$$

There fore , the UPPER 95 % LIMIT of B_q life is

$$x_o e^{+\frac{1.645}{b\sqrt{N(.5 + .5q)}}} \qquad (q \leq .5)$$

NUMERICAL EXAMPLE

From a data set of 12 items there is 1 suspended prior to the 10% failure level, and

Weibull slope = b = 1.5

Estimated B_{10} life = x_o = 100 Hours .

QUESTION : What are the Lower 5% and Upper 95% Limits on B_{10} life ?

SOLUTION

Central Estimate of B_{10} life = 100 Hours = x_o

$$\text{Lower 5\% Limit} = x_o\, e^{-\frac{1.645}{b\sqrt{N(.5 + .5(.1))}}}$$

In this case, b = 1.5 , N = 12 -1 =11, q= .1 . Therefore , the Lower 5% Limit on B_{10} life becomes

$$100\, e^{-\frac{1.645}{1.5\sqrt{11(.55)}}} = 64 \text{ Hours}$$

The Upper 95% Limit on B_{10} life becomes

$$100\, e^{+\frac{1.645}{1.5\sqrt{11(.55)}}} = 156 \text{ Hours}$$

Thus, we are 90% confident that the true population B_{10} life is somewhere between 64 hours and 156 hours .

CONCLUSION

It can be seen how 90% Confidence Bands can be constructed at any quantile level q when we know

(1) The Weibull slope at Level q

(2) The Sample Size at Level q

(3) The Central Estimate of B_q life on the Weibull plot of the data set

NOTE : These are Nominal Limits obtained by using the Median Standard Deviation of the Logarithm of B_{10} life .