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DETROIT RESEARCH INSTITUTE
P.O. BOX 36504 • GROSSE POINTE, MICHIGAN 48236 • (313) 886-7976

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

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A MONTE-CARLO PROGRAM FOR GENERATING RANDOM SAMPLES FROM A NORMAL POPULATION

INTRODUCTION

It often becomes desirable to generate random samples in a Monte-carlo fashion from a statistical population whose parameters are known. This is especially true in situations where market or consumer data are not readily available and we still would like to know the range of variation present in a given marketing or customer sample whose size is dictated by sales rates and the elapsed time since the start of a product's release. In light of this need for random samples in many industrial programs dealing with product variability, we are presenting this statistical bulletin which will outline a Monte-Carlo procedure for sampling out a **Normal Population** with a known **Mean** and **Standard Deviation**.

THE EVIDENCE APPROACH TO THE PROBLEM

The most powerful procedure for dealing with questions of samples and their related populations is the procedure known as the **EVIDENCE APPROACH.** The mathematical definition of **Evidence** that a measurement is outside a given population is

EVIDENCE = Natural Logarithm of ODDS ,

where the ODDS for a measurement is defined by the ratio

$$\text{ODDS} = \frac{\text{Probability that Measurement is Outside the Population}}{\text{Probability that Measurement is Inside the Population}}$$

A WEIBULL EXAMPLE OF EVIDENCE

Assume that we encounter a measurement x which is so large that it is in an upper tail of a population. If the population is a **Weibull Distribution** with **Slope b** and **Characteristic Value θ**, then the **Evidence** that the measurement is too large to be in the population so defined is given by the formula

$$\text{EVIDENCE} = E(x) = \ln[e^{(x/\theta)^b} - 1]$$

For example, if the Weibull population has $b = 2$ and $\theta = 5$ and we encounter an $x = 12$, we calculate $E(12) = \ln[e^{(12/5)^2} - 1] =$ 5.757 units of EVIDENCE of being out of the population. Therefore , the ODDS of 12 not belonging to the Weibull population is $e^{E(12)} = 316$ to 1 .

APPLYING THE EVIDENCE APPROACH TO NORMAL POPULATIONS

In the case of a Normal population we have the two parameters

M = Mean of population
Sigma = Standard Deviation of population
(Also called the Sigma parameter)

For such a Normal population the EVIDENCE for a large measurement x being outside the population is defined by the formula

$$E(x) = (\pi/\sqrt{3})(\text{Z-Score for } x) ,$$

where Z-Score for x = (x - M)/sigma .

Thus , $E(x) = [\pi/\sqrt{3}][(x - M)/\text{sigma}]$.

From this EVIDENCE FORMULA we conclude that the ODDS for concluding that x is outside the Normal population is

$$\text{ODDS} = e^{[\pi/\sqrt{3}][(x - M)/\text{sigma}]}.$$

A NUMERICAL EXAMPLE FOR A NORMAL POPULATION

Suppose we are dealing with a Normal population with {M = 10; Sigma = 2}.

Suppose we encounter a measured value x = 15. The EVIDENCE for being an OUTLIER is $E(15) = [\pi/\sqrt{3}][(15 - 10)/2] = 4.534$ units of EVIDENCES.

So, ODDS $= e^{4.534} = 93$ to 1 in favor of concluding that x = 15 is too large to be in the NORMAL POPULATION with Mean = 10 and Sigma = 2 .

GENERATING RANDOM NORMAL MEASUREMENTS

The logical way to generate random measurements from a population is to select a bunch of RANDOM NUMBERS representing PROBABILITIES between 0 and 1, and then locating these probabilities on the EVIDENCE SCALE (from $-\infty$ to $+\infty$) within the Normal Distribution taken as the underlying population.

Let F = RND = A Random Number between 0 to 1 (a probability)

Then , EVIDENCE = E = ln[F/(1 - F)] .

For a NORMAL POPULATION : E(x) = [$\pi/\sqrt{3}$][(x - M)/Signma]

So, **x = M + [($\sqrt{3}$ Sigma E)/π]** .

Thus, we have generated a RANDOM VALUE x from the assumed NORMAL POPULATION with the specified MEAN and SIGMA . By this technique we are able to write a complete Monte-Carlo computer program for generating any size random sample we desire from a NORMAL population. A typical computer printout from such a program is shown on page 5. For further information contact DETROIT RESEARCH INSTITUTE.

MONTE-CARLO SAMPLING FOR THE NORMAL DISTRIBUTION

(Also called the Evidence Model)

POPULATION MEAN = 20
POPULATION SIGMA = 5
TRIAL NUMBER = 1
SIZE OF SAMPLE DESIRED = 40
RANDOM SEED NO. SELECTED = 29

THE SAMPLE VALUES (X) ARE LISTED BELOW:

X = 12.18417
X = 22.54817
X = 16.07087
X = 16.37031
X = 17.19842
X = 21.82332
X = 25.74354
X = 21.65997
X = 17.80113
X = 20.56877
X = 25.25982
X = 16.73806
X = 17.94333
X = 14.28985
X = 15.08159
X = 21.85482
X = 17.33909
X = 21.5481
X = 26.37686
X = 20.11644
X = 17.11583
X = 28.12689
X = 19.53565
X = 18.1062
X = 31.93277
X = 21.20356
X = 17.71677
X = 33.06213
X = 20.68167
X = 18.27773
X = 20.91678
X = 16.46347
X = 19.948
X = 23.85032
X = 22.15724
X = 19.51957
X = 29.87608
X = 19.27829
X = 15.38952
X = 27.61199

SAMPLE MEAN = 20.73218

SAMPLE SIGMA = 4.720516