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HOW TO CONSTRUCT CONFIDENCE INTERPOLATION CHARTS FOR ENTROPY RATIOS BETWEEN TWO SAMOLES

INTRODUCTION

In the world of design reliability we are always confronted with questions of confidence with respect to reliability improvements, i.e., product improvements durability-wise. This simply amounts to a comparison of failure tendencies in any designs being compared. If a new design has fewer failures (i.e., needs fewer repairs) in a specific service time, then we say the new design is better, i.e., more reliable or more durable for that service time. The number of failures per system in a given service time is what is called the Entropy of the system for that service time. Consequently, what is needed is a chart showing the confidence that we can have in any desired Entropy Ratio, i.e., any desired ratio of repairs in one system versus another. This bulletin addresses itself to this very question.

THE BASIC APPROACH TO CALCULATING CONFIDENCE

Every question concerning confidence indices in reliability statistics can be answered by making use of the Universal Law of Odds, which states that

$$\text{ODDS} = \text{ENTROPY RATIO}^{\text{ODDS EXPONENT}}$$

If, for example, we have two system designs for a given application in service with different failure rates and, consequently, different maintenance frequencies of repair, we make log-log plots of both by taking Service Times as abscissas and Failures per system as ordinates, as in Figure 1:

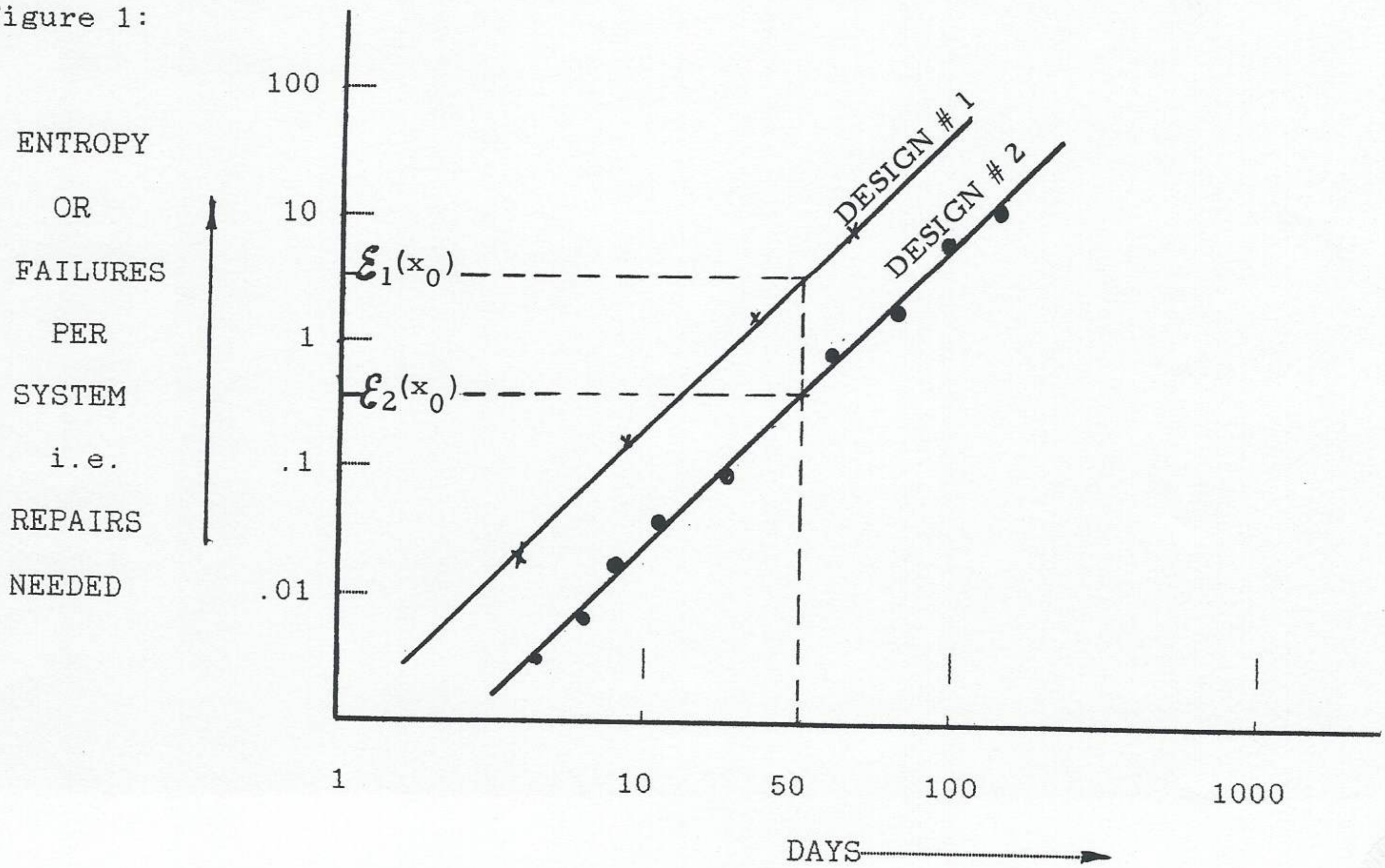


FIGURE 1

Note that the needed number of repairs in any service time (say 50 days) is what is called Entropy at that service time, or, what is the same thing, the number of failures per system in that service time.

Now, suppose that at a special service time x_0 (say 50 days) the Entropy, i.e., number of repairs needed by Design #1 is $\mathcal{E}_1(x_0)$ while the number of repairs needed by Design #2 is $\mathcal{E}_2(x_0)$, where $\mathcal{E}_2(x_0) < \mathcal{E}_1(x_0)$, thus indicating that Design #2 is superior to Design #1.

The question raised by this situation is "What is the confidence that Design #2 is better than Design #1 in service period x_0?".

In order to calculate this confidence we compute the Odds in favor of Design #2 over Design #1 for x_0 days.

We know the Entropy Ratio $\mu = \mathcal{E}_1(x_0)/\mathcal{E}_2(x_0)$ and the

$$\text{ODDS EXPONENT is } \eta = \frac{\sqrt{1 + \sqrt{N1 * N2}/.5(N1 + N2)}}{.55[1/\sqrt{N1*(.5 + .5Q1)} + 1/\sqrt{N2*(.5 + .5Q)}]}$$

Where N1 = Size of the sample for Design #1

and N2 = Size of the sample for Design #2

$Q1 = 1 - \exp[-\mathcal{E}_1(x_0)]$ (Q1=<.5) : $Q2 = 1 - \exp[-\mathcal{E}_2(x_0)]$ (Q2=<.5)

NOTE: In case Q1>.5 then replace Q1 by 1 - Q1.

Also, in case Q2>.5 then replace Q2 by 1 - Q2.

CONFIDENCE CORRESPONDING TO SPECIFIC ODDS

Since Odds = C/(1 - C) (C = Confidence)

It follows that C(Confidence) = Odds/(1 + Odds).

From this formula we can determine the desired Confidence for any such problem.

A NUMERICAL EXAMPLE

Suppose in Figure 1 we have

N1 = Design #1's Sample Size = 4 .

N2 = Design #2's Sample Size = 9 .

$\mathcal{E}_1$(50 Days) = 2 Failures on Design #1 in 50 Days

$\mathcal{E}_2$(50 Days) = .8 Failure on Design #2 in 50 Days.

Then, μ = Entropy Ratio = 2/.8 = 2.5

Furthermore, from the formula for the Odds Exponent : η = 2.389475

So, ODDS = $(2.5)^{2.389475}$ = 8.93

Therefore, the Confidence that Design #2 is better than Design #1 is

C = CONFIDENCE = 8.93/9.93 = .90

CONSTRUCTING THE CONFIDENCE INTERPOLATION DIAGRAM

For an Entropy Ratio => 1 we have Odds = 8.93 (C = .90)

For Entropy Ratio => 1.5 we have Odds = $(2.5/1.5)^{2.3895}$ = 3.3893

For a Confidence (C) of 3.3893/4.387893 = .77

For Entropy Ratio => 2 we have Odds = $(2.5/2)^{2.3895}$ = 1.7043

For a Confidence (C) of 1.7043/2.7043 = .63

For Entropy Ratio => 2.5 we have Odds = $(2.5/2.5)^{2.3895}$ = 1.00

For a Confidence (C) of 1/2 = .50

These Confidence Indices for these Entropy Ratios are plotted in Figure 2 (called the Confidence Interpolation Diagram).

COMPUTER OUTPUT FOR THE NUMERICAL EXAMPLE

(ENTROPY COMPARISON PROGRAM)

Desired Life = 50

1st Sample Size = 4

2nd Sample Size = 9

1st Entropy = 2.00

2nd Entropy = 0.80

Obs. Entropy Ratio = 2.50

Median Odds Exponent = 2.38947492

Median Odds = 8.93033336

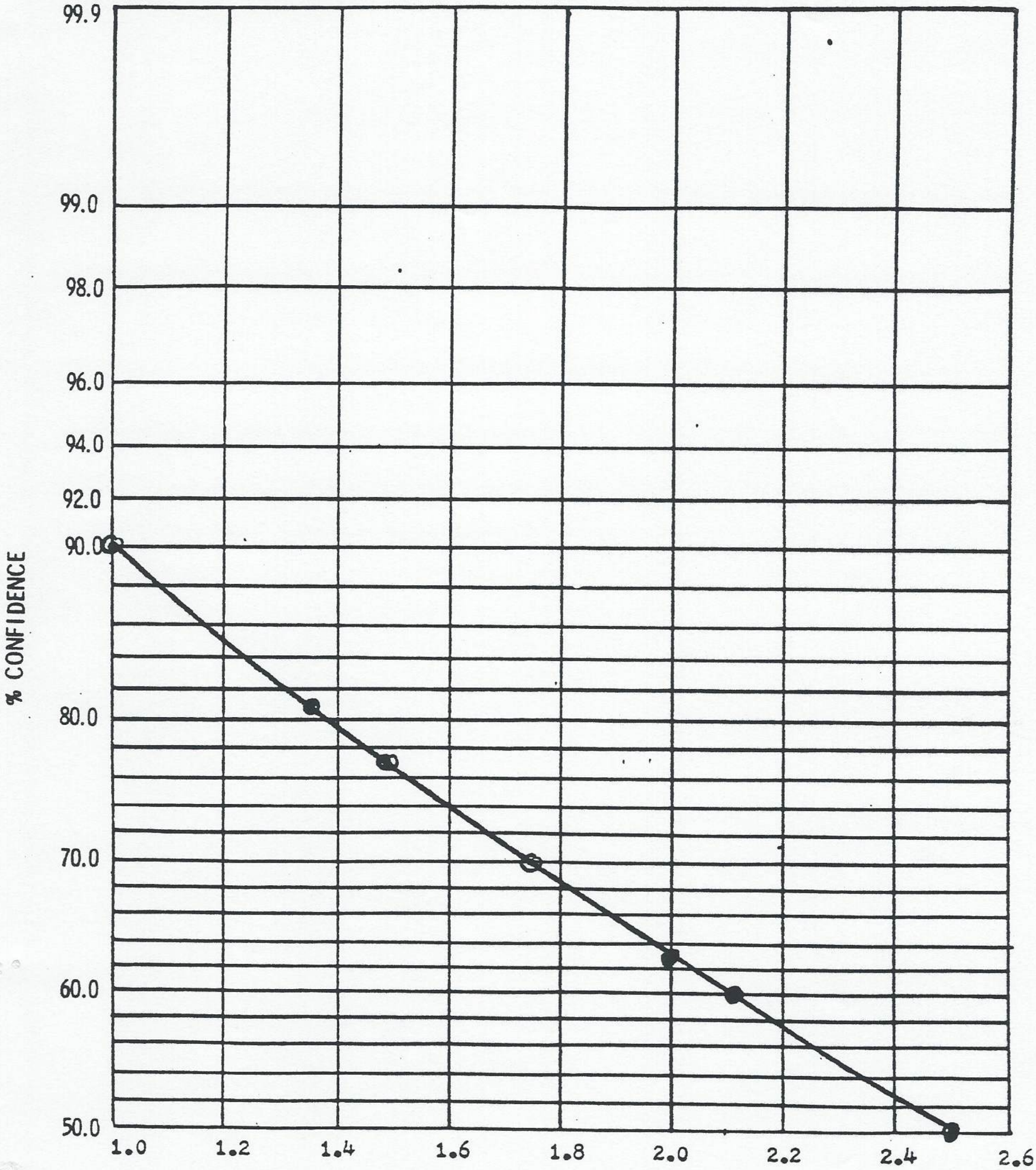
Confidence for Unit Null Ratio = 0.899298446

Confidence for Null Ratio 1.375 = 0.806670067

Confidence for Null Ratio 1.750 = 0.701042237

Confidence for Null Ratio 2.125 = 0.595881798

FIGURE 2 --- CONFIDENCE INTERPOLATION CHART FOR ENTROPY RATIOS



ENTROPYY RATIO = $\frac{\text{50 day FAILURE RATE OF DESIGN \# 1}}{\text{50 day FAILURE RATE OF DESIGN \# 2}}$