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# THE COMMON SENSE BASIC JUSTIFICATION FOR WEIBULL THEORY

## WHY THIS BULLETIN WAS SUGGESTED

We daily come across all kinds of questions being asked by curious thinkers who are not satisfied with taking for granted someone else's assumed statistical approach to a particular problem. This is especially true about the Weibull approach to failure problems in components and systems designed and built by modern industry. This becomes very critical when the procedures used can affect customer satisfaction for manufactured products both as far as convenience and reliability in operation are concerned. In many areas competition is so keen that the slightest slip-up or defect can have serious repercussions on the success of a manufacturing business. With these factors in mind we are presenting a list of basic questions and answers with regard to the now widely used Weibull approach to problems of product failure and reliability over service usage time.

## ONE DOZEN QUESTIONS AND ANSWERS

QUESTION #1: What is the universally troublesome fact with regard to systems?

ANSWER #1: The universally troublesome thing about systems is that no system is perfectly free of breakdowns or defects indefinitely.

QUESTION #2: What are the basic types of breakdowns or defects from the standpoint of service quality?

ANSWER #2: The basic types of breakdowns or defects are

I Repairable Defects
- (a) Those requiring down time to fix
- (b) Those which heal themselves while the system is in use.

II Irreparable Defects
- (a) Micro-defects which, though irreparable, do not prevent system operation, but do shorten the life of a system by progressive deterioration.
- (b) Terminal failures which put an end to any further possible operation of a system.

QUESTION #3: On a service time scale what is always true about a system?

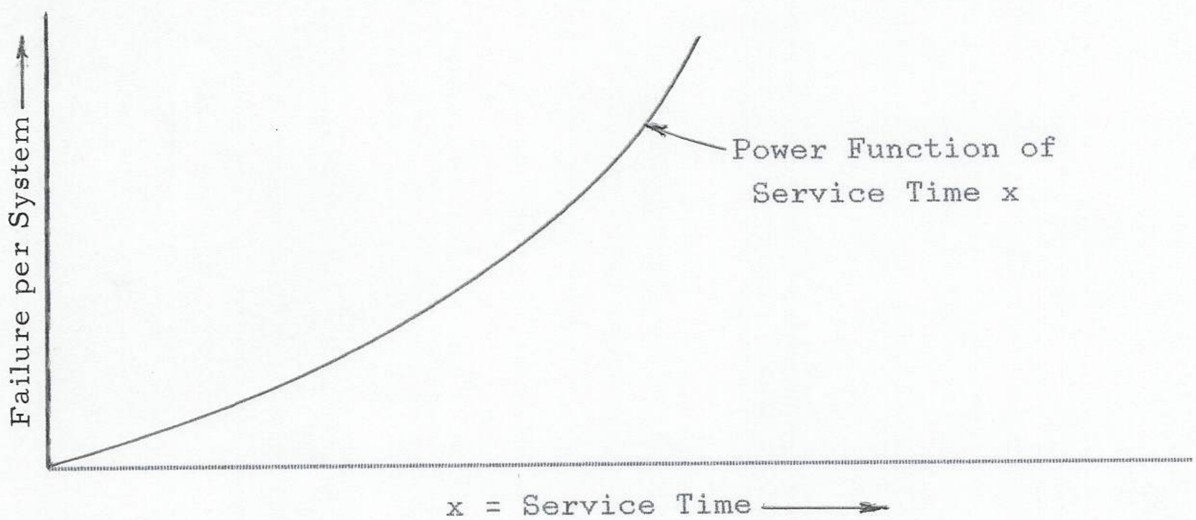
ANSWER # 3: A system always shows an increasing number of defects (breakdowns) as service time increases. In other words, the longer a system is used the higher the accumulated repair total (failures per system) becomes.

QUESTION #4: What characterizes the Weibull approach to system life studies?

ANSWER # 4: The Weibull method of system life studies assumes repairs (failures per system) increase cumulatively as some power of service time. Graphically, this can be pictured as follows:

Weibull Law

Failure per System in Time x = CONSTANT * $x^b$




QUESTION #5: If the cumulative number of failures per system increases linearly (directly proportional) with time in service, what can be said of the system's life distribution ?

ANSWER #5: If the accumulated failure total for a system increases linearly with usage time then the system life distribution is exponential, which implies that the average time breakdowns (failures) is constant.

QUESTION #6: How can the Weibull approach be simplified graphically ?

ANSWER #6: The Weibull outlook (using power functions of service time for cumulative failure totals for systems) can be simplified graphically by using Log-Log graph paper in order to make a power function of service time x become a LINEAR relation between the **Log of x and the Log of Cumulative Repairs per System**. This is the essence of **Weibull Probability Paper.**

QUESTION #7: Since repairs per system is a purely **dimensionless** number, how can we express the power formula

$$\text{Failure per System} = \text{Constant} * x^b$$
$$(x = \text{service life})$$

as to make the formula dimensionless?

ANSWER #7: We can express the formula as follows:

$$\text{Failures per System (In Service Time } x) = (x/\Theta)^b$$

Where $1/\Theta^b$ is the **Constant** by which $x^b$ is multiplied.

QUESTION #8: If a population of a certain type of system shown an average of $\mathcal{E}(x)$ failures per system in time x, what fraction of the population will show no failures in that time x?

ANSWER #8: The fraction of the population which is failure free (reliable) to service time x is exp $[-\mathcal{E}(x)]$.

QUESTION # 9: What is the stochastic (statistical) name for the number of failures (breakdowns) of a system in service time x?

ANSWER # 9: The statistical name given to failures (breakdowns) per system in service time x is the **ENTROPY at x**.

QUESTION #10: What characterizes a **Weibull Distribution** of failures in system life studies?

ANSWER # 10: A Weibull distribution of system life is characterized by the fact that

**FAILURES PER SYSTEM IN SERVICE TIME x**
**= ENTROPY at service time x = $(x/\Theta)^b$**

QUESTION #11: What is the name of the exponent b in the Weibull model?

ANSWER # 11: The exponent b is known as the **Weibull Slope**.

QUESTION #12: What is the name given to the quantity $\Theta$ in the Weibull model where

Failures per System (in service time x) = $(x/\Theta)^b$?

ANSWER # 12: The quantity $\Theta$ in the Weibull model has been given the name **Characteristic Life**.

## FURTHER DISCUSSION OF THE WEIBULL MODEL

As a supplement to the preceding dozen of questions and answers, let us now go into a discussion between a STUDENT and his instructor with regard to **WEIBULL ANALYSIS**.

STUDENT: It has been generally observed that **Fatigue Failures** in simple specimens such as Beams or Bearing Elements do plot as genuine Weibull distributions with linear plots on Weibull paper. How do you explain this?

INSTRUCTOR: Fatigue is caused by irreparable types of micro-defects which were mentioned in answering Question #2. Fatigue is a progressive breakdown of the cohesive bonds inside of material specimens. Generally, these micro-separations or breakdowns cannot be repaired once they get started, but, instead, these deteriorations grow continually until, finally, there is a **MACRO SEPARATION** called **FATIGUE**. The prevalent micro-separations increase in size proportional to some power function of stress cycles, and, as such, have Weibull life distributions of cycles to the final Macro -Separation called Fatigue.

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STUDENT: Is there a way of looking at failures in general, and how does the Weibull Model fit into the GENERAL MODEL?

INSTRUCTOR: Any system subject to failure by various modes can be analyzed by observing the **Number of Failures** (breakdowns) the system experiences with increasing service time. The quantity (FAILURES PER SYSTEM), in accumulated service time x , we called the ENTROPY at service time x. The probability of a system failing (at least once) by the time the ENTROPY reaches a certain level [ say $\mathcal{E}(x)$ ] is then F(x), where

$$F(x) = 1 - \exp[-\mathcal{E}(x)] .$$

The Weibull Model is a special case of the General Model, in which

$$\mathcal{E}(x) = (x/\Theta)^b$$