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LEONARD G. JOHNSON
EDITOR

DETROIT RESEARCH INSTITUTE
P.O. Box 36504 • Grosse Pointe, MI 48236 • (313) 886-8435

WANG H. YEE
DIRECTOR

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DESIGN OF LIFE TESTS - (DOLT)

INTRODUCTION

As is true of any computerized programs, so it is the case with the design of life tests, namely, that there is both **input** and **output.** We have adopted the acronym **DOLT** to represent our computerized Design of Life Tests. In this bulletin we outline the input steps and output steps of the program and show an actual example to illustrate the entire Life Testing Design Program. By means of this straightforward procedure the Design of Life Tests becomes scientifically standardized without the usual ambiguities involved in dealing with such projects.

INPUT ITEMS

(THE WORSE CASE SCENARIO APPROACH-----FOR MAXIMUM PRODUCT ASSURANCE)

1 How many items will be sold?
(This is the Sales Total T.)

2 What is item's acceptable life?
(This is the Goal Life x_0)

3 How many failed items can be tolerated among the total sold in the acceptable life period?
(This is the Number Defective D.)

4 What is the profit per sold item?
(This is the Gain Factor G.)

5 What is the loss per failure?
(This is the Loss Factor L.)

6 By what factor do you want profits to exceed losses?
(This is Profitability Factor K.)

OUTPUT ITEMS

1. Quantile Level to be studied:
   [This is $Q = (D + 1) / (T + 1)$.]
   (Goal $B_q = x_o$)

2. Required Odds for the Goal B_q:
   (Odds Required = KL/G)

3. What is Test B_q from a test sample of N items?
   (Find this from sample Weibull plot at quantile level **q**)

4. What is the test confidence with respect to Goal B_q?
   [Use Life Ratio (Test B_q/Goal B_q) in DRI's Computer Program "**GOALCNF**" to find test confidence $\mathbf{C_{test}}$.)

5. Compare $\mathbf{C_{test}}$ with $\mathbf{C_{req}}$, where
   $C_{req} = KL/(G + KL)$.
   Continue testing until $\mathbf{C_{test} > C_{req}}$.
   Use Sequential Analysis. As long as $\mathbf{C_{test} < Creq}$ ***the product is unacceptable.***

A TYPICAL EXAMPLE

Input Items

1. A particular engine models is going to have a sales volume of $T = 10{,}000$.

2. The acceptable life is $x_o = 50{,}000$ Miles before any **Major Failure** .

3. We want **None** of the 10,000 engines sold to have a **major failure** before 50,000 Miles. Thus, $D = 0$.

4. The **Profit** per sold engine is $G = \$75$.

5. The **Loss** (replacement cost) of a **major** failure before 50,000 miles is $L = \$600$.

6. The manufacturer wants profits from sales to be at least twice all replacement losses due to major failures before 50,000 miles,i.e., the Profitability Factor is $K = 2$.

OUTPUT ITEMS

1. The quantile level under study is
   $Q = (0 + 1)/(10{,}000 + 1) = 0.0001$

2. Odds Required $= 2(600/75) = 16/1$
   (For B_q Life at least 50,000 Miles)
   Thus , $C_{req.} = 16/17 = .9412$

3. Four engines are tested to a major failure:
   (Mileages: 589,100; 793,120; 959,400; and 1,158,000)
   The Weibull plot of these 4 values has a slope of 3.5 and
   $B_{.0001} = 69{,}800$ Miles.

4. The life ratio at quantile **q** is
   Test B_q/Goal $B_q = 69{,}800/50{,}000 = 1.396$
   So, Test Odds $= (1.396)^{\text{Odds Exponent}}$
   where, Odds Exponent $= \pi(3.5)\sqrt{[4(1 + .0001)/6]} = 8.978$

   Thus, Test Odds $= (1.396)^{8.978} = 19.989$
   and, Test Confidence $= C_{test} = 19.989/20.989 = .9524$

5. Since the Test Confidence (.9524) exceeds the
   Required Confidence (.9412), we conclude that all is **OK.**

THE MATHEMATICS OF "GOALCNF" PROGRAM

Test Odds = $(\text{Life Ratio})^{\text{Odds Exponent}}$

Life Ratio = Test B_q/Goal B_q

Odds Exponent = $\pi\, b \sqrt{[N(1 + q)/6]}$

b = Weibull slope of Test Weibull plot
q = Quantile Level
N = Test Sample Size at Quantile **q**
= Total Tested - No. Suspended Prior to **q.**

C_{test} = Test Odds/(1 + Test Odds)

"GOALCNF" PRINTOUT FOR THE EXAMPLE

GOAL CONFIDENCE PROGRAM

QUANTILE LEVEL = .0001
BQ GOAL AT QUANTILE LEVEL = 50000
SAMPLE WEIBULL SLOPE = 3.5
SAMPLE BQ LIFE = 69800
SAMPLE SIZE AT BQ LIFE = 4
CONF. OF MEETING BQ GOAL = .9526868

CONCLUDING REMARKS

After entering in the input items for the example, we determined the output values and found that after one test of four items to failure the test confidence exceeded the required confidence. This meant that we could accept the product as one which would comply with our profitability desire of having the profitability factor = 2. It had turned out that the test confidence was less than the required confidence of .9412, we would have continued testing with another sample and then combined two confidence indices into a resultant test confidence and then we would have compared this resultant with the required confidence of .9412. For example, suppose the first test sample gave only .82 as its test confidence, and then this was followed by another test sample having .85 as its test confidence.

The Resultant Test Confidence would be
$(.82)(.85)/[(.82)(.85) + (.18)(.15)] = .697/.724 = .9627$, which exceeds the required confidence of .9412, and thus tells us that the product is acceptable for meeting the profitability factor which was originally specified in the input. It can be seen how systematically handy this whole approach is.

APPENDIX

Possible Modifications of the <u>**DOLT**</u> Program:

1. Instead of the worst case scenario, which assumes failure implies that all sold items are bad, we can consider **Loss** as being **Loss Per Sold Item,** with an estimated fraction, say 20%, or 1 in 5, being unable to survive the life goal. Let us use the symbol F_0 for the fraction of sold items which could be bad. **Then** the formula for the **Odds Required** in a life test of compliance to the B_q **Goal** becomes

$$\textbf{Odds Required} = F_0KL/G \quad .$$

2. If, in addition to **Regular Major** failures and their losses, there is a **Zooming** type of **Catastrophic Loss** of magnitude Z, then the **Required Odds** becomes

$$\textbf{Odds Required} = K\{[(Z/T) + F_0L]/G\}$$