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THE EASY WAY TO HANDLE PROBABILITIES IN LIFE TESTS (A SURE-FIRE WAY TO ASSURE PRODUCT DURABILITY)

INTRODUCTION

If you want to become versatile at performing some scientific task, learn the fundamentals behind the task. This is true in any skillful activity which an individual desires to master. It certainly is true in the field of life testing, where it is of utmost importance to evaluate the durability of a product design with regard to its reliability to perform satisfactorily for a long enough time in actual service usage. In this bulletin we shall introduce the marvels of all the information which can be fished out of Weibull plots of life test data and the natural probability facts they contain.

The Inherent Destroyer of Systems ----- Entropy

What are the odds in favor of a product living up to a promised life goal or guaranteed warranty period? This is the critical question which must be answered in every life testing experiment which has as its objective the prediction of what length of service life can be expected when the item under study is put to use by customers. This means that we must find a way of measuring the growth of weakness (or aging) in the product with respect to service time. There is throughout all of nature a concept which specifically relates to the aging or wear-out process in any system. This is the concept of **Entropy**. The famous Weibull Distribution is specifically tied into the concept of Entropy due to the fact that Weibull Probability Paper has the **Logarithm of Entropy** as its vertical scale. What this mean is that Weibull assumed that Entropy grows as some power of service time, and consequently, it serves as a convenient representation of any aging process vs. operating time at any particular period in the usage of a product which becomes more vulnerable with increasing age.

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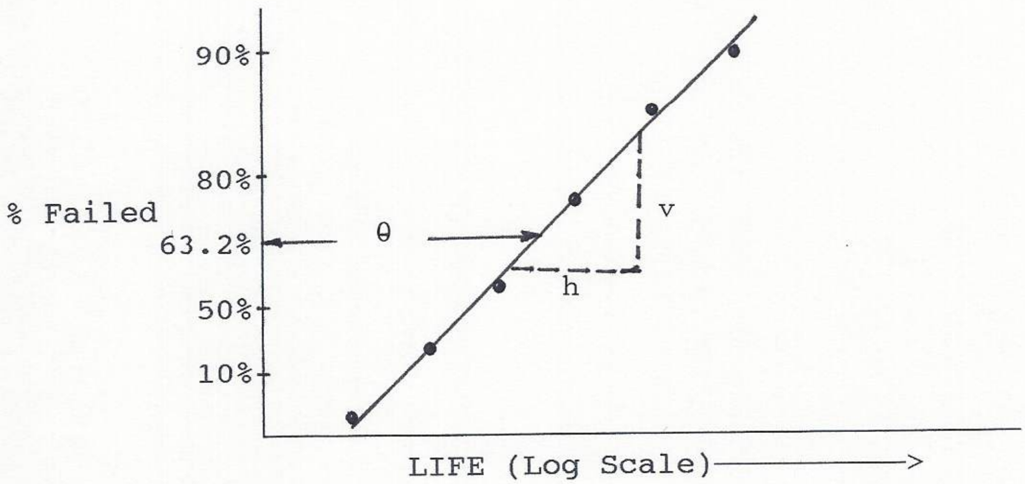
Weibull Plots and Life Testing Probabilities

Weibull plots of life test data are constructed by assigning each failed item a median plotting position in the entire distribution of failure times which have been arranged into numerical order, from the shortest life to the longest life. These plotting positions are known as **Median Ranks**, as estimated by **Benard's Formula (J - .3)/(N + .4)**, where J = order no., and N = sample size. For a Weibull Distribution the plot thus constructed on Weibull Probability Paper will produce a straight line with a particular slope (known as Weibull Slope) and a location characterizing life length. **b** and θ are the **symbols** for the two parameters involved in such straight line plots on Weibull Paper.

θ = Characteristic Life (at 63.2% failed)

b = Weibull Slope = v/h



Odds in Favor of Compliance With a Life Goal
(The Easy Way to Find The Answer)

The fundamental problem in any durability compliance determination is the calculation of the odds in favor of living up to a guaranteed life promised to the customer. To begin with, in order to have more than 50% chance of complying, the sample we are testing must show a life above the desired goal. The odds in favor of compliance increases as the life ratio **(Test Life/Goal Life)** is made larger. The ratio of test life to goal life is called the ***Durability Safety Factor.*** The more that test life exceeds goal life, the safer we are in guaranteeing compliance with the goal life.

Odds in favor of compliance also depends on the test sample size, as well as the Weibull slope. When we talk about **"Test Life"**, we are referring to the same particular quantile level, which identifies the **"Goal Life"**, e.g., B_{10} life, where 10% of the population is supposed to fail. In general we can deal with any **Bq** life, where **q** is the fraction failed (quantile level).

The easy way to calculate the odds in favor of compliance with a B_q life goal is to use the **Universal Law of Odds,** given by the formula

$$\textbf{Odds} = (\textbf{B}_q \textbf{ Life Ratio})^{\textbf{Odds Exponent}}$$

where, B_q Life Ratio = (Test B_q Life/Goal B_q Life) ,

and, Odds Exponent $= \pi\, b\sqrt{(N/6)(1 + q)}$ $(q \leq .5)$

or $\pi\, b\sqrt{(N/6)(2 - q)}$ $(q > .5)$

b = Weibull slope, q = Quantile Level

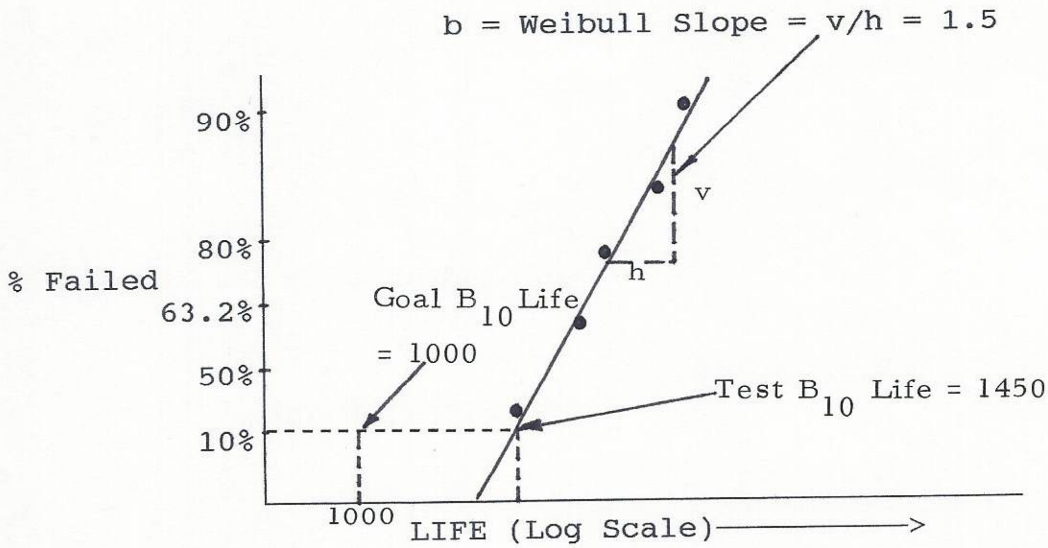
N = Sample Size at Quantile Q = Total Tested - No. Unfailed prior to q

This entire mathematical procedure is automated in the computer program "GOALCNF", which gives a final answer as a confidence define by

$$\textbf{Confidence} = \textbf{Odds / (1 + Odds)}$$

A Numerical Example

Below is shown a Weibull plot of 5 items run to failure, and exhibiting a Weibull slope of 1.5 and a B_{10} Life of 1,450 hours. The warranty goal for B_{10} Life is 1,000 hours. Thus, b = 1.5, q = .1, and N = 5.



B_q Life Ratio = B_{10} Life Ratio = 1,450/1,000 = 1.45

Odds Exponent = $\pi(1.5)\sqrt{(5/6)(1 + .1)}$ = 4.512

So, Odds = $(1.45)^{4.512}$ = 5.347 to 1

Confidence = 5.347/6.347 = .842

The printout of "GOALCNF" for this same problem is shown on page 7

GOALCNF PROGRAM

QUANTILE LEVEL = .1
BQ GOAL AT QUANTILE LEVEL = 1000
SAMPLE WEIBULL SLOPE = 1.5
SAMPLE BQ LIFE = 1450
SAMPLE SIZE AT BQ LIFE = 5
CONF. OF MEETING BQ LIFE GOAL = .8429654

Conclusion

The critical question of compliance confidence with respect to B_q life goal is easily determined when the data Weibull plot is compared to a life goal at any specified quantile q.