STATISTICAL BULLETIN
Reliability & Variation Research

LEONARD G. JOHNSON
EDITOR

DETROIT RESEARCH INSTITUTE
P.O. Box 36504 • Grosse Pointe, MI 48236 • (313) 886-8435

WANG H. YEE
DIRECTOR

Volume 24
Bulletin 1

April, 1994
Page 1

JUDGING DESIGN IMPROVEMENT BY SEQUENTIAL COMPARISON TESTS UNTIL SUFFICIENT PROFITABILITY IS DEMONSTRATED

INTRODUCTION

How do you accept a life improvement promise on a new design? If someone asked you that question, what would your answer be? The first thing you should ask before you could answer any question about life improvement is "What is meant by *life?* The word *life* by itself is meaningless quantitatively until we specify its *quantile level,* or *percentile level,* in the distribution of lives to which it belongs. For example, we might be specifically interested in the length of life at which 10% of a population will have failed (commonly called the B_{10} Life). Suppose, for example, that you have a new design of which you run a *life test* on five specimens to failure, with the following results (put into numerical order) :

256 Hrs., 495 Hrs., 701 Hrs., 1,105 Hrs., 1,499 Hrs.

Suppose, furthermore, that you wanted to compare this new design to an old design which you tested some months ago with six specimens which gave lives to failure as follows:

166 Hrs., 340 Hrs., 490 Hrs., 750 Hrs., 1,090 Hrs., 1,520 Hrs.

How would you determine the confidence that the new design can be accepted as having a better $B_{10\%}$ Life than the old design? It must be admitted that this is an important question which must be carefully answered before we can proceed to adopt the new design as a replacement for the old design. It is this type of comparison problem which we shall discuss in the subsequent pages of this month's bulletin.

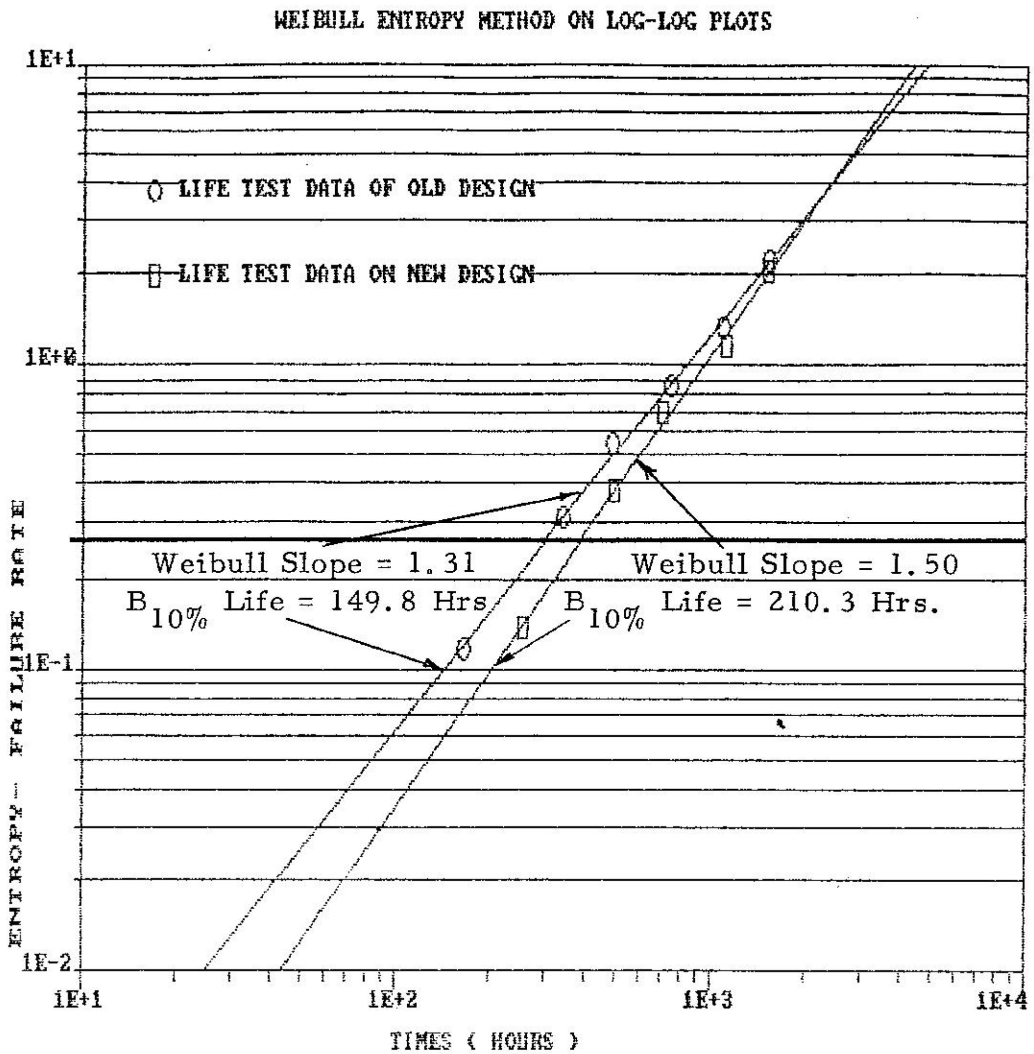
STEP # 1 ----- MAKE WEIBULL PLOTS OF THE DATA SETS

The first thing we must do is make Weibull plots from the data sets in the life tests of the two designs. In order to do this we form tables of the life values which consist of each life in one column and the Corresponding Median Rank or Entropy for that life in a second column. Doing this, we obtain the following two tables for the two designs we are comparing:

Entropy $= \xi(x) = \ln(1/1\text{-}F)$

OLD DESIGN			NEW DESIGN		
Life	Median Rank	Entropy	Life	Median Rank	Entropy
166 hrs.	.1094	.1159	256 hrs.	.1296	.1388
340	.2656	.3087	495	.3148	.3780
490	.4219	.5480	701	.5000	.6931
750	.5781	.8630	1105	.6852	1.1588
1090	.7344	1.3260	1499	.8704	2.0433
1520	.8906	2.2130			

Now we are ready to plot these data sets on **Weibull Probability Paper** or **Weibull Entropy Paper (log-log paper)**. We chose to use the log-log paper because it is readily available on any graphic software with a log-log plotting routine. The plots for the two designs are shown on page 3.



THE REGRESSION POLYNOMIAL OF LINE 1 -

(-3.825E+00) + (1.309E+00)*X
THE VARIANCE - 4.869E-04

THE REGRESSION POLYNOMIAL OF LINE 2 -

(-4.453E+00) + (1.496E+00)*X
THE VARIANCE - 5.822E-04

STEP # 2 ----- ENTER THE WEIBULL DATA INTO "**BQCNF**" PROGRAM

We next take the information obtained from the two Weibull plots and enter it as inputs into the computer program called "**BQCNF**". When we do this, we obtain the following type of printout:

BQ LIFE CONFIDENCE PROGRAM

Quantile Level = .1
1st Weibull Slope = 1.31
1st B_q Life = 149.8
1st Sample Size = 6
2nd Weibull Slope = 1.5
2nd B_q Life = 210.3
2nd Sample Size = 5

CONFIDENCE THAT 2ND B_q > 1ST B_q = .7426449

Thus, we see that the confidence we are seeking about the superiority of the new $B_{10\%}$ life over the old $B_{10\%}$ life is slightly over 74% confidence. If we desire more confidence, we must do some more testing by taking another sample of each design and coming up with another comparison for another confidence index to be superimposed on top of the first one (.7426449). We continue such **Sequential Testing** until we arrive at the confidence level dictated by **Gains** and **Losses**, as well as the desired **Profitability Ratio**. For illustration of these concepts with these two designs, we go to the next section of this bulletin.

STEP # 3----- DOING FURTHER TESTING OF TWO DESIGNS TO INCREASE CONFIDENCE

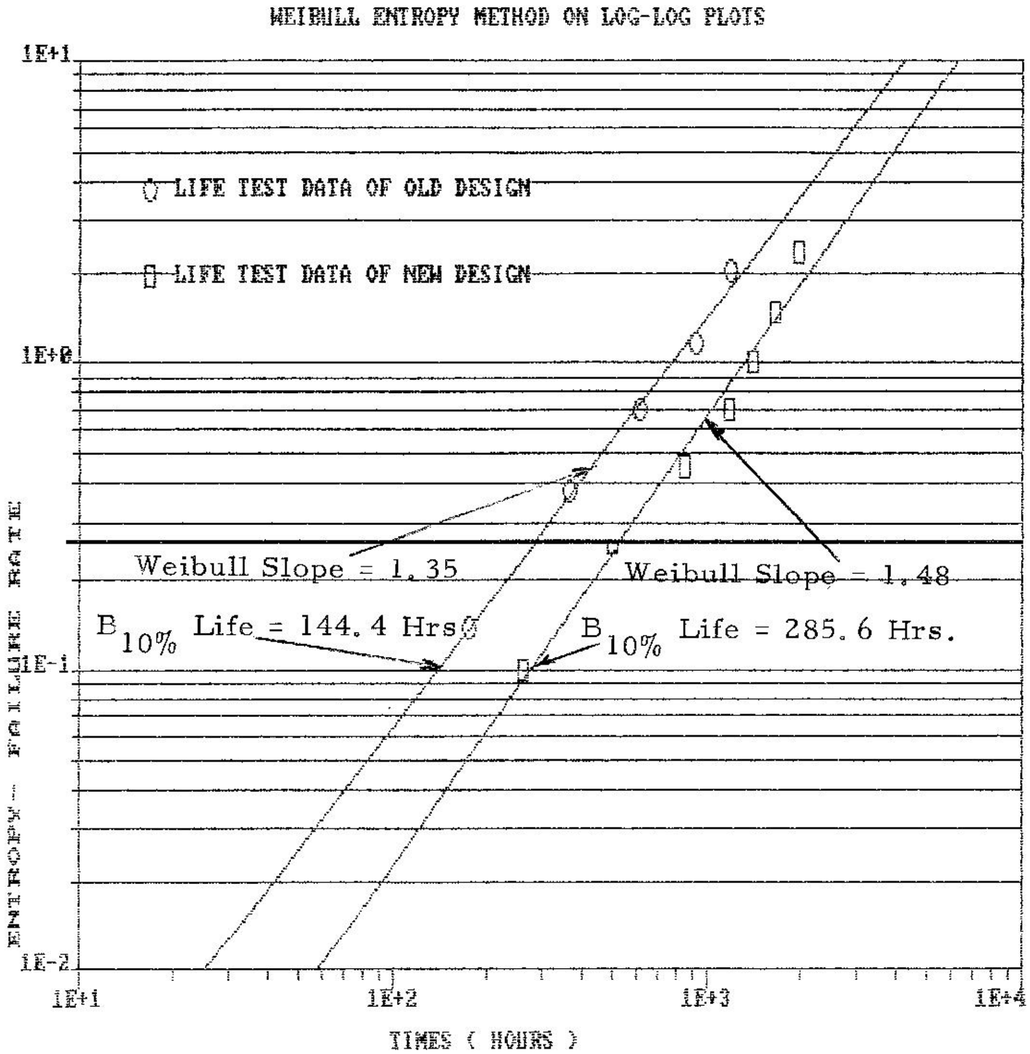
Suppose we adopt the new design and that we gain 2.5 million dollars in case it really is superior to the old design at $B_{10\%}$ life. But, in case it does not turn out to give a longer B_{10} life we stand to lose an estimated 12 million dollars. What this means is that just to break even we need **Odds** = 12/2.5 or 4.8 to 1 in favor of superiority at B_{10} life in the new design. However, let's say we want to gain **twice** as much as we could lose in the long run. Then, the **Odds Required** become **twice** as great, i.e., 9.6 to 1. This means that we need to prove in our testing program that we have **Evidence** = ln(9.6) = 2.26176 units of evidence for an improved $B_{10\%}$life. From the first comparison test which we ran we had a **Confidence** = .7426449, which is equivalent to **Odds** equal to the quotient .7426449/.2573551 = 2.88568. From this **Odds,** we get **Evidence** = ln(2.88568) = 1.05976 units of evidence. So it can be seen that we need further testing for enough evidence to reach our **Profitability** objective of getting **twice** as much **Gained** as **Lost.**

We next take, let us say, a sample of 5 specimens of the old design and 7 specimens of the new design. These sample sizes are what we happened to have readily available. After running these specimens to failure we obtain the following tabulated data sets:

Entropy = $\xi(x)$ = ln(1/1 - F)

OLD DESIGN			NEW DESIGN		
Life	Median Rank	Entropy	Life	Median Rank	Entropy
175 hrs.	.1296	.1388	261 hrs.	.0946	.0994
365	.3148	.3780	501	.2297	.2610
612	.5000	.6931	850	.3649	.4540
925	.6852	1.1558	1179	.5000	.6931
1195	.8704	2.0433	1402	.6351	1.0081
			1655	.7703	1.4710
			1976	.9054	2.3581

The Weibull plots for these two data sets are shown on the next page.



THE REGRESSION POLYNOMIAL OF LINE 1 -

(-3.885E+00) + (1.347E+00)*X
THE VARIANCE - 1.139E-03

THE REGRESSION POLYNOMIAL OF LINE 2 -

(-4.602E+00) + (1.476E+00)*X
THE VARIANCE - 4.084E-03

We take the information from the above Weibull plots and enter the relevant factors into the program "**BQCNF**" to come up with the following printout:

B_q CONFIDENCE PROGRAM

Quantile Level = .1
1st Weibull Slope = 1.35
1st B_q Life = 144.4
1st Sample Size = 5
2nd Weibull Slope = 1.48
2nd B_q Life = 285.6
2nd Sample Size = 7

CONFIDENCE THAT 2ND B_q > 1ST B_q = .9010648

Thus, we have a second confidence number of .9010648 that the new design has a better $B_{10\%}$ life than the old design. This amounts to additional **Odds** of .9010648/.0989352 = 9.10763 to 1, or additional **Evidence** = ln(9.10763) = 2.20911 units of evidence. Therefore, altogether we now have a **Total Evidence** equal to the sum 1.05976 + 2.20911 = 3.26887 units of evidence. Since this exceeds the required evidence of 2.26176 units, we conclude that we can be sufficiently confident of the new design's superiority at $B_{10\%}$ life.

CONCLUSION

From the discussion we have presented in this bulletin regarding the problem of comparing two sample Weibull plots, as we did with the example of a new design versus an old design, we can clearly draw the following important conclusions:

Conclusion # 1: The computer program "BQCNF" of DRI's CARS Software is very handy when we want to come up with a **Confidence Index** for the hypothesis that a new design has a better B_q Life than some previous design.

Conclusion # 2: Even though the first comparison test we run on two designs does not give enough confidence to promise a sufficiently large **Profitability Factor**, we can still do some more comparison tests to come up with enough **Additional Evidence** in favor of a new design. Then we simply add together all the units of evidence from separate comparisons to obtain the **Total Evidence** that the new design is better at the B_q Life than the old design. If the new design is really good enough to be economically feasible and with the desired profitability, then performing additional comparison tests will bring this out.