STATISTICAL BULLETIN
Reliability & Variation Research

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Volume 25
Bulletin 1

April, 1995
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REALISTIC MINIMUM RELIABILITY IMPLIED BY SUCCESS RUNS OF VARIOUS SIZES

INTRODUCTION

A success run in a life test to a specified life target (bogey) is defined to be a sample of life test specimens all of which have survived to the bogey life. Obviously, the larger the success run sample size is, the more we can guarantee in the way of a minimum reliability, i.e., a worst condition.

The purpose of this bulletin is to come up with a table of minimum guaranteed reliabilities for different magnitudes of success runs. This will enable test engineers to arrive at reasonable success run sample sizes required for any desired reliability level at a specified confidence level. This is something which is greatly needed because the classical approach gives such outlandish sample sizes for required success runs, all due to the realistic assumption that the minimum population reliability could be zero. Obviously, with a success run to its credit, a product cannot have zero reliability!

THE BASIC MATHEMATICS INVOLVED

Given N successes to target x, we reason as follows:
With N successes and no failures, a system being tested is still in the random stage of its life, where the Weibull slope is unity.

We take a fictitious extra item and make the conservative assumption that it fails exactly at target x. Thus, we have 1 failure in N + 1, all of which have been run to target life x. Thus, at target x, the entropy per failure is $(N + 1)(x/\theta)$, where θ is the Characteristic Life, which for unit Weibull slope is the *Mean Time Between Failures.*

Since the Median Entropy to a failure is *ln2,* it follows that the Median Characteristic Life $\theta_{.50}$ must be such that

$$(N + 1)(x/\theta_{.50}) = \ln 2$$

$$\text{or} \quad (x/\theta_{.50}) = \ln 2/(N + 1) = \textit{Median Entropy at } x$$

Thus, the estimated Median Reliability to target x is

$$R_{.50}(x) = \exp[-\ln 2/(N + 1)] = .5^{(1/N+1)}$$

Then, the Minimum Reliability to target x is

$$A = R_{MIN}(x) = \left[.5^{\left(\frac{1}{N+1}\right)} \right]^{\exp\left[\frac{6}{.5(N+1)}\right]}$$

This actually represents the *Lower 6 Sigma Limit* in a Log-Normal distribution of Entropy.

Thus by taking the Minimum Reliability to be **A**, we can write the revised formula for Reliability with Confidence **C** as follows:

$$R_c(x) = A + (1 - A)(1 - C)^{\frac{1}{N+1}}$$

Example: If N = 10, then

$$A = \left(.5^{\frac{1}{11}} \right)^{\exp(6/\sqrt{5.5})} = .93893^{12.91525} = .44316$$

TABLE OF MINIMUM RELIABILITIES FOR SUCCESS RUNS

SUCCESS RUN SAMPLES	MINIMUM RELIABILITY (LOWER 6-SIGMA LIMIT)
5	2.495329E-02
6	8.658472E-02
7	.1754715
8	.2717081
9	.3626618
10	.4431557
11	.5122065
12	.5706491
13	.6199151
14	.6614888
15	.6967034
16	.7266852
17	.7523612
18	.7744823
19	.7936552
20	.8103706
30	.9024492
40	.9383662
50	.9563859
60	.9668842
70	.9736295
80	.9782713
90	.9816317
100	.984161

CONCLUSION

By taking the lower 6-sigma limit as our basis for minimum reliabilities in success run testing, we have arrived at realistic values for minimum levels of reliability in such tests. These minimum values of reliability can then be employed as inputs into the **Compressed Success Run** formula to predict reliability at any desired confidence level.