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USING THE Z-SCORE STATISTICAL APPROACH IN ORDER TO ESTIMATE THE WORST CASE OF POSSIBLE DEVIANT BEHAVIOR ON THE LOW SIDE OF A PREDICTED B-Q LIFE FROM A LIFE TEST DATA SET FITTED BY A TWO-PARAMETER WEIBULL PLOT

INTRODUCTION

For any phenomenon subject to variation it becomes necessary to exercise control of situations which may arise whenever deviant behaviour patterns occur. This is true of any dimensional specs on manufactured items, required tolerance limits in nutrition or pharmaceutical doses, product durability variations in life tests, strength requirements in mechanical parts or structures, as well as studies involving the human mind and body in psychiatry.

The way in which deviant behavior is usually measured is to determine how many standard deviations a measured criterion or score deviates from the accepted normal or average value considered to be nominal. The crazier a deviant behaviour is, the more sigmas or standard deviations it is away from the average. This number of sigmas (defining deviant or outlying behaviour) is called a **Z-Score**. The greater the Z-Score, the more convincing is the evidence of a deviant behavior.

In this bulletin we shall introduce a new computer program with the name ZSIGMALF, by which we will be able to pinpoint any extreme value of life in a life test for any quantile level (such as B-10) by simply specifying the Z-score we want to calculate for that quantile level of life. In this way we can come up with the worst life to be expected in those cases where we want to specify a minimum reliability level for purposes of designing proper life tests with proper sample sizes as determined from proper prior distributions used in compressed success run tests or other types.

SPECIFYING THE ESSENTIAL ELEMENTS NEEDED FOR A Z-SCORE

The Z-Score for a deviant value in a distribution is defined as follows:

Z = (Deviant Value - Nominal Value)/Sigma .

For the case of B-Q life study, we work with natural logarithm of the B-Q life, and the corresponding sigma (standard deviation) of the logarithm of the B-Q life The common practice in life testing to failure (with perhaps some items unfailed, i.e., suspended) is to construct a Weibull plot of the data using **Median Ranks.** Then the Nominal Value of B-Q life is read as the abscissa corresponding to the fraction failed, i.e., Q, on the vertical axis of Weibull paper. In order to determine the **Sigma** of ln(B-Q life), we use the Weibull Slope from the data plot, as well as the **Sample Size at Quantile Q**, and, also, the value of Q itself. Knowing these factors, we can calculate the **Sigma** of ln(B-Q life) from the following formula:

SIGMA = 1/{b*SQR[.5*N*(1 + Q)]} ,

where b = Weibull Slope of the data plot
N = Sample Size at B-Q Life

Note: The correct formula for N is the following:

N = NO. SUSPENDED PRIOR TO THE B-Q LIFE .

Since we are dealing with logarithms, we define the Z-Score as

Z = [log(deviant B-Q) - log(nominal B-Q)]/Sigma
= b * SQR[.5*N*(1 + Q)] * log(deviant B-Q/nominal B-q)

Note: The symbol log denotes the natural (Naperian) logarithm .
In those cases where Q > .5 **we redefine** Q as Q' = 1 - Q .

EVIDENCE AND CONFIDENCE LEVELS FOR DEVIANT BEHAVIOUR

Evidence of deviant behaviour is mathematically defined to be the natural logarithm of the **Odds Against Normal Behaviour**. It so turns out that **Evidence** of deviant behaviour can be conveniently represented as $Z(\pi/\sqrt{3})$. Then it will follow that the **Confidence** of deviant behaviour is

$$C = \text{Confidence} = 1/[1 + \exp(-\text{Evidence})]$$

$$= 1/\{1 + \exp[-Z(\pi/\sqrt{3})]\}$$

$$= 1/[1 + \exp(-1.8138Z)]$$

__Note:__ In case the value of Z is negative, we must use its absolute (positive value) in calculating the confidence of deviant behaviour on the low end of the distribution of measured values being studied.

For example, at a Z-Score of $Z = -3$, the confidence of deviant behaviour is

$$C = 1/[1 + \exp(-5.4414)] = 0.995685 .$$

A TYPICAL EXAMPLE TO ILLUSTRATE THE PROCESS

Suppose we are given a set of life testing data which consists of 10 specimens run to failure, and that these 10 data points (lives) plot out on Weibull paper into a line with a Weibull Slope b = 1.5, and a Characteristic Life = 1000 hours. Then that same plot will show 10% failed (i.e., a B-10% Life) at

$$1000\,[\ln(1/9)]^{(1/1.5)} = 223.076 \text{ Hours} \quad .$$

Furthermore, since N = 10, and b = 1.5, and Q = 0.1, we calculate the

$$\text{SIGMA} = \frac{1}{1.5\sqrt{0.5 \cdot 10(1 + 0.1)}} = .2842676$$

Now, the value of ln(nominal B-10% life) = ln(223.076) = 5.4075125 .

If we want the lower -3 **Sigma** value for the B-10% life, we would evaluate it by subtracting 3 * Sigma, i.e., 3 * 0.2842676 or 0.8528028, from 5.4075125.

This yields 5.4075125 - 0.8528028 = 4.5547097 .

Then, the low value of B-10% life (at the Z-Score of Z = -3) becomes exp(4.5547097) = 95.079 Hours.

AUTOMATING THE CALCULATIONS BY USING THE COMPUTER PROGRAM "ZSIGMALF"

The calculations we made on page 4 can be automatically done by going to the computer program "ZSIGMALF", which is an acronym for "Z-SIGMA-LIFE", in which life is understood to be at some quantile level Q, which is under study. For the example on page 4 we would get the following computer printout after telling the program to take a Z-SCORE = -3, together with a Weibull Slope = 1.5, and a sample size of N = 10, and quantile level Q = 0.1 :

ZSIGMALF PROGRAM

CALCULATING THE Z-SIGMA BOUND OF B-Q LIFE

DESIRED Z-SCORE = -3
DESIRED QUANTILE LEVEL = 0.1
MEDIAN VALUE OF LIFE AT THE QUANTILE LEVEL = 223.076
WEIBULL SLOPE = 1.5
SAMPLE SIZE AT QUANTILE LEVEL = 10
- 3 - SIGMA VALUE OF B-10 LIFE = 95.07915

CONCLUSION

We have demonstrated in this bulletin how to come up with a low side deviant value of any B-Q life in a two parameter Weibull population by making use of the new computer program called "ZSIGMALF", in which we specify what Z-SCORE we want to define as being our measure of deviant behaviour, as well as specifying the nominal value of B-Q life from a Weibull line, together with the Weibull Slope of the line, and the **sample size** at quantile level Q which is under consideration. The calculated value of the Z level of B-Q life can then be employed as a reference value for the purpose of predicting worst case scenarios with regard to the durability and life reliability of the design being manufactured and eventually sold to the public. This will also be of great help in coming up with proper test sample sizes in various types of life testing programs, be they **Success Run Tests**, or complete or partly complete tests to failure. In this way the test engineer can avoid the excessively large sample sizes dictated by classical approaches in which **Zero Life** is considered to be the lower limit. It is much more reasonable to take a Z-SCORE of -3, or perhaps -6, as being representative of the worst possible deviant behaviour on the low end of the B-Q life distribution at quantile Q.