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THE APPLICATION OF EQUIVALENCE THEORY IN ACCELERATED TESTING PROGRAMS

INTRODUCTION

Modern industry is constantly plagued by short lead times in product development and testing programs. For this reason it has become necessary to speed up or accelerate testing schemes so as to make it possible to meet deadlines imposed by a competitive world in the selling of consumer products. Accelerating or shortening a test is a science in itself, governed by its own basic laws. The most important of these laws governing accelerated life tests is the Law of Equivalence of test plans as dictated by the Equivalence Theorem which states that test plans are equivalent if, and only if, the average entropy per failure is the same for each test. This fundamental law can be applied in many different ways to design accelerated life tests. Among the various situations in which this Equivalence Theorem can be applied are:

I: Success Run Tests to different targets without any stress change.

II: Success Run Tests to the same targets, but at different stress levels.

III: General Life Tests which show either a failure or success for each individual test item at its own running time.

SUCCESS RUN TESTING

EXAMPLE #1: Suppose that at a certain load (stress) we are required to run 100 specimens without any failing to a target of 1,000 hours. If the Weibull slope for this failure phenomenon is 2.5, how many specimens would be required to last 2,000 hours without any failures in order to be equivalence to 100 successes to 1,000 hours, as originally proposed?

SOLUTION: We use the Universal Sample Size Theorem for Accelerated Testing, which states that

$$\text{SAMPLE SIZE RATIO} = \frac{1}{\text{ENTROPY RATIO}} \qquad (1)$$

For this example : Entropy Ratio $= \dfrac{\text{Entropy at 2000 Hours}}{\text{Entropy at 1000 Hours}}$

$$= \frac{\left(\frac{2000}{\theta}\right)^{2.5}}{\left(\frac{1000}{\theta}\right)^{2.5}} = \left(\frac{2000}{1000}\right)^{2.5} = 2^{2.5} = 5.657$$

(θ = Characteristic Life at Test Stress)

Thus, by Equation (1) above:

$$\text{SAMPLE SIZE RATIO} = \frac{\text{SAMPLE SIZE FOR 2000 HRS.}}{\text{SAMPLE SIZE FOR 1000 HRS.}} = \frac{1}{5.657},$$

$$\text{OR, SAMPLE SIZE FOR 2000 HRS.} = \frac{100}{5.657} = 18.$$

Thus, 18 Successes to 2,000 Hrs. would be equivalent to 100 Successes to 1,000 Hrs., i.e., 100,000 specimen hours can be reduced to 36,000 specimen hours by using the Equivalence Theorem expressed by Equation (1).

EXAMPLE #2: Suppose that the Characteristic Life of a certain design in a fatigue test varies inversely as the 5th power of stress.

Now, suppose that a reliability spec for the design with a Weibull slope 3 requires that we successfully run 200 items for 1,000 hours at stress 100,000 psi. How much can we reduce the sample size 200 by increasing the stress to 120,000 psi., and still running the specimens for 100 hours without any of them failing?

SOLUTION: Use the Universal Sample Size Theoem for Accelerated Testing, which states that

$$\text{Sample Size Ratio} = \frac{1}{\text{Entropy Ratio}} .$$

In this case,

$$\text{Entropy Ratio} = \frac{\left(\dfrac{1000}{\theta_2}\right)^3}{\left(\dfrac{1000}{\theta_1}\right)^3} = \left(\frac{\theta_1}{\theta_2}\right)^3$$

{θ_1 = Characteristic Life at Stress #1 (100,000 psi)}
{θ_2 = Characteristic Life at Stress #2 (120,000 psi)}

$$\frac{\theta_1}{\theta_2} = \left(\frac{120,000}{1000,000}\right)^5 = (1.2)^5 = 2.48832$$

So, Entropy Ratio = $(2.48832)^3$ = 15.407

$$\text{Sample Size Ratio} = \frac{\text{Sample Size for } 120,000 \text{ psi}}{\text{Sample Size for } 100,000 \text{ psi}} = \frac{1}{15.407} = 0.0649 .$$

Thus, Sample Size for 120,000 psi = 200(.0649) = 13.

Thus, 13 specimens running successfully for 1,000 hours each at 120,000 psi is equivalent to 200 specimens running successfully for 1,000 hours at 100,000 psi. This is a reduction of 200,000 specimen hours down to 13,000 specimen hours.

BOGEY TESTS WITH BOTH SUCCESSES AND FAILURES

EXAMPLE #3: Suppose we run enough specimens for 1,000 hours until there are two of them which fail in 1,000 hours. The size of sample that must be run in order to realize an average reliability of 98% is **N** , such that N - 2 + 1/N + 2 = .98, or N-1/N+2 = .98, from which we obtain

$$N - 1 = .98N + 1.96$$
$$\text{or} \quad .02N = 2.96$$
$$\text{or} \quad N = 148$$

Thus,, 146 Successes and 2 Failures will meet the required average reliability of 98% to a target of 1,000 hours.

Now, suppose the test stress is increased by 10%, and that the S-N Exponent is K = 5. Then the specimen life is divided by the factor $(1.1)^5 = 1.61$.

Therefore, in *x* hours the Entropy Function is $\left[\frac{x}{\left(\frac{\theta_1}{1.61}\right)}\right]^b = (1.61)^b\left(\frac{x}{\theta_1}\right)^b$

{θ_1 = Characteristic Life at Original Stress and b = Weibull Slope}

Thus, the new Entropy (at 10% extra stress) is $(1.61)^b$ times as large as it was under the original stress for the same target *x*.

Now, suppose the Weibull slope is b = 3. Then the Entropy to any target *x* becomes multiplied by $(1.61)^3 = 4.17$, i.e., there will be 4.17 times as many failures per system when the stress is raised by 10%. This means that, whereas, originally, we had an Entropy total $148(1000/\theta_1)^3 = 2$, implying that $\theta_1 = 4{,}198$ Hrs. (the Characteristic Life at original stress). Now, at 10% extra stress we have an Entropy total of $148(1000/\theta_2)^3 = 2(4.17) = 8.34$ Failures in 1,000 hours in a sample of 148 items. This makes $\theta_2 = 2{,}608$ Hrs. as the Characteristic Life under the higher stress. Now, in order to have only 2 failures in 1,000 hours at the higher stress we must select a sample size N_2 such that $N_2(1000/2608) = 2$, or $\underline{0.056374N_2} = 2$, or $N_2 = 36$.

Thus, at 10% more stress we need only 36 specimens for a 1,000 hour test in order to get 2 failures. Thus, we get by with 36,000 specimen hours instead of 148,000 specimen hours. Again, this shows the validity of the formula

$$\text{Sample Size Ratio} = \frac{1}{\text{Entropy Ratio}}$$

CONCLUSION

It is truly remarkable how the concept of statistical predictions of product durability under various testing conditions can be readily handled by the Universal Law of Sample Sizes for accelerated testing programs. This universal law can be simply stated as follows:

$$\text{SAMPLE SIZE RATIO} = \frac{1}{\text{ENTROPY RATIO}} \qquad (1)$$

The way to employ this law is to determine the Entropy levels for two testing schemes and then dividing the Accelerated Test Entropy to its target by the Standard Test Entropy to its target to obtain the Entropy Ratio. Then by using Equation (1), i.e., the Universal Sample Size Law, we can determine how to modify the standard sample size (for the unaccelerated test) to obtain the sample size for the accelerated test.