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PARTIALLY DEBITTED RENEWAL THEORY

INTRODUCTION

In previous bulletins (July 1977 and April 1978) we discussed FULLY DEBITED RENEWAL THEORY and FULLY NON-DEBITED RENEWAL THEORY, respectively. In this bulletin we shall discuss INTERMEDIATE CASES, namely, PARTIALLY DEBITED RENEWAL THEORY. We must use partially debited renewal theory whenever the age of a machine PARTIALLY AFFECTS the performance of a brand new component which is installed in the place of a failed component.

Obviously, the result we get from partially debited theory must fall between what is obtained using fully debited theory on the one hand and fully non-debited theory on the other hand. In the discussion which follows we shall show how this "In Between" , or PARTIALLY DEBITED situation, is to be handled in making predictions on failures per machine when we are studying the failures of a component in the machine in those cases where the age of the machine partially degrades the replacement component.

DRI STATISTICAL BULLETIN

October , 1978

Volume 8 Bulletin 5

THE TWO EXTREMES AND THE "IN BETWEEN" SITUATIONS IN RENEWAL STUDIES

FIGURE 1 below shows the various situations we might face in studying FAILURES PER MACHINE on LOG-LOG PAPER, when we are dealing with a part which has Weibull Parameters (b, θ) , and which is repeatedly installed as a replacement for the failures of the same part in a set of one or more similar machines.

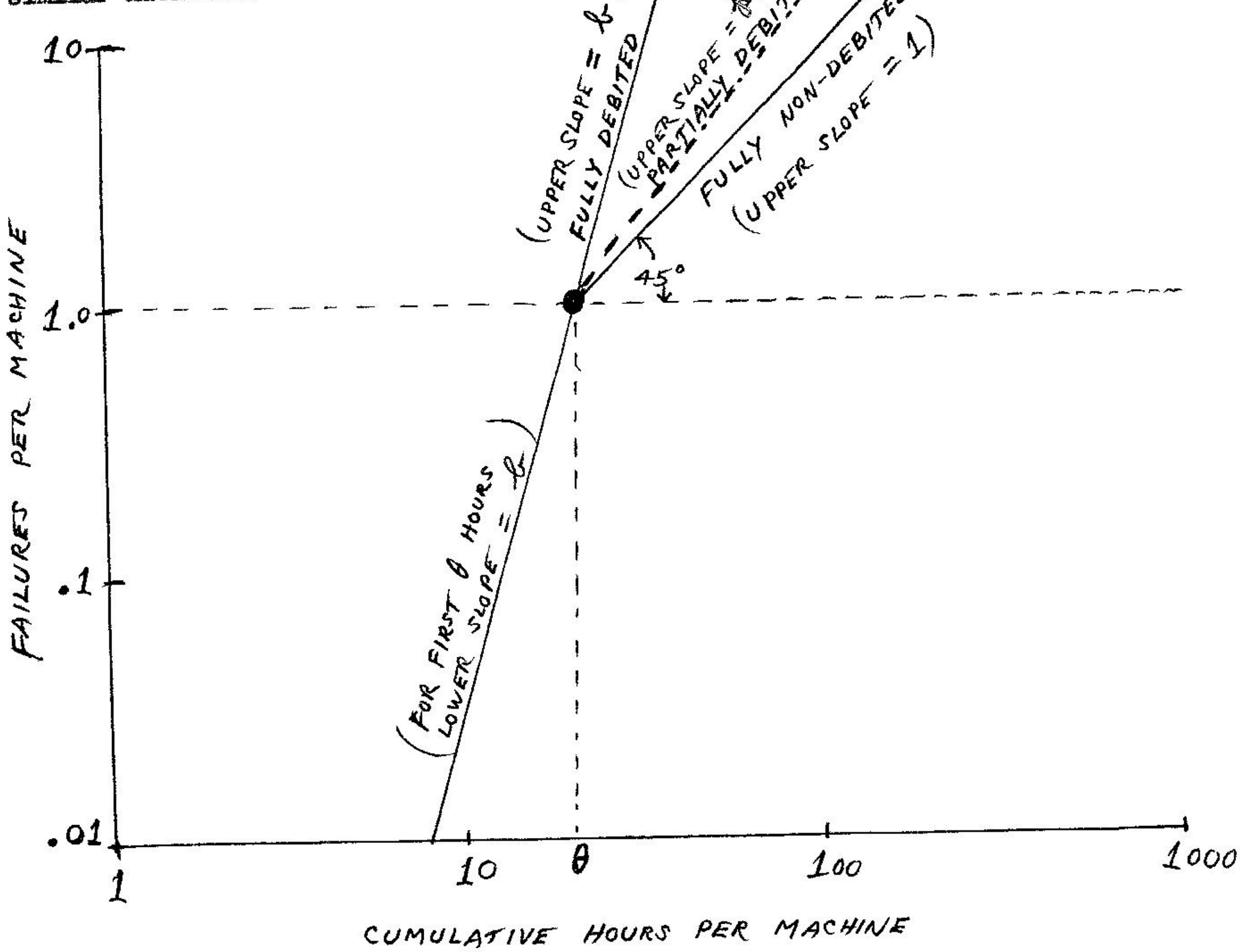


FIGURE 1

DRI STATISTICAL BULLETIN
Volume 8 Bulletin 5 October , 1978

DISCUSSION OF FIGURE 1

In FIGURE 1 we see a part which has WEIBULL SLOPE b and CHARACTERISTIC LIFE θ . This part is installed in a machine and the failures of the part in the machine are plotted versus the cumulative hours on the machine using LOG-LOG paper. In case FULLY DEBITED THEORY applies, the complete graph of failures per machine vs. hours per machine is simply ONE STRAIGHT LINE of SLOPE b , and passing through an ordinate of 1 FAILURE PER MACHINE at $X = \theta$ hours.

In case of FULLY NON-DEBITED THEORY the UPPER PORTION of the graph (i.e., beyond θ hours) goes up at a 45° angle, i.e., the UPPER SLOPE , beyond θ hours , is UNITY .

Finally, in the case of PARTIALLY DEBITED RENEWAL THEORY the graph to the right of the first θ hours goes between the fully debited upper slope b and the fully non-debited upper slope 1. The indicated slope $pb + q$ is simply a WEIGHTED AVERAGE of the two slopes b and 1.

p is some positive number between 0 and 1, and $q = 1 - p$, i.e., $p + q = 1$.

In case $p = .5$ and $q = .5$ we say the part failures are SEMI-DEBITED.

In case $p = .75$ and $q = .25$ we say the part failures are 75% debited.

In case $p = .25$ and $q = .75$ we say the part failures are 25% debited.

In case $p = .1$ and $q = .9$ we say the part failures are 10% debited.

In case $p = .9$ and $q = .1$ we say the part failures are 90% debited.

In case $p = 1$ and $q = 0$ we say the part failures are fully (100%) debited.

In case $p = 0$ and $q = 1$ we say the part failures are fully non-debited, i.e., 0 % debited.

AN EXAMPLE

Suppose a part is repeatedly installed as a replacement into machines of a certain type, and suppose the part's failures cause the FAILURES PER MACHINE of that part to grow with CUMULATIVE HOURS PER MACHINE in accordance with TABLE I below:

TABLE I

CUMULATIVE HOURS PER MACHINE	TOTAL MACHINES	TOTAL FAILURES OF PART
20	833	2
40	769	30
60	690	136
80	550	343
100	405	527
125	275	625
150	200	717
175	145	764
200	130	957
400	19	791

We construct the following table of FAILURES PER MACHINE vs. HOURS PER MACHINE by dividing the failure total by the machine total for each number of hours per machine.

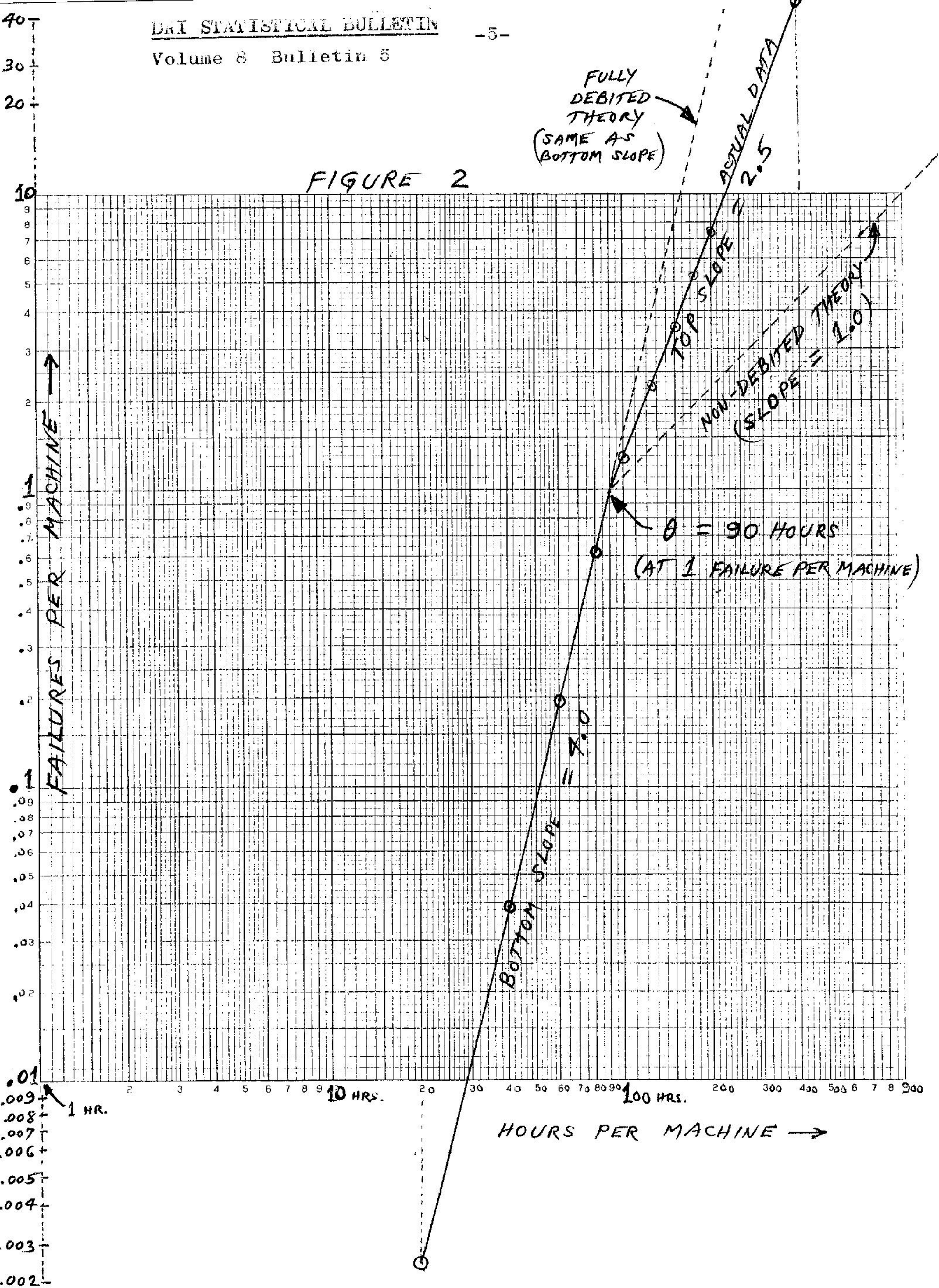
TABLE II

HOURS PER MACHINE	FAILURES PER MACHINE
20	.0024
40	.0390
60	.1971
80	.6236
100	1.301
125	2.273
150	3.585
175	5.269
200	7.632
400	41.63

Now we plot the results in TABLE II on LOG-LOG PAPER by putting HOURS PER MACHINE on the abscissa scale and FAILURES PER MACHINE on the ordinate scale.

THE RESULT IS FIGURE 2 shown on page 5.

FIGURE 2


FULLY DEBITED THEORY (SAME AS BOTTOM SLOPE)
ACTUAL DATA
TOP SLOPE = 2.5
NON-DEBITED THEORY (SLOPE = 1.0)
θ = 90 HOURS (AT 1 FAILURE PER MACHINE)
BOTTOM SLOPE = 4.0
FAILURES PER MACHINE →
HOURS PER MACHINE →
1 HR.
10 HRS.
100 HRS.

CONCLUSIONS FROM FIGURE 2

We see that the graph of the actual data in FIGURE 2 crosses 1 FAILURE PER MACHINE at X = 90 hours. Hence, for the part in question

$$\theta = 90 \text{ hours (Characteristic Life)}$$

Furthermore, the bottom part of the graph has a slope of 4.0. Hence, for the part under study: WEIBULL SLOPE = b = 4.0.

The upper part of the graph has a slope of 2.5. Hence, we can find the factor p from the equation

$$p\,b + q = 2.5 .$$

Using the initial slope b = 4.0 , and putting q = 1 - p , this equation becomes

$$4p + 1 - p = 2.5$$

$$\text{or} \quad 3p = 1.5$$

$$\therefore \quad p = .5$$

Hence the failures of this part are SEMI-DEBITED !