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THE RANDOM RATIO TECHNIQUE OF PREDICTION

I : INTRODUCTION

Since predicting invariably reduces to a scheme of extrapolating past and present trends into the future , it is only natural to inquire what is a convenient method of doing this . There is a randomness in future behaviour which must be accounted for by means of one or more random variables in the prediction scheme . The type of functional relation chosen for the prediction scheme must be consistent with the nature of the distribution functions of the random variables . In this bulletin we shall discuss a technique which employs a random ratio between the values of a transient function at times t and λt , where $0 \leq \lambda \leq 1$.

II : ANALYTICAL RELATIONS

Let $Y(t)$ denote the value of a MONOTONE INCREASING function of time at the time t . Such functions arise in economic studies whenever we consider such items as cumulative demand , cumulative sales , cumulative costs , etc. Let $Y(\lambda t)$ denote the value of the same monotone increasing function at some previous time λt . where $0 \leq \lambda \leq 1$.

Now suppose

$$Y(t) = \mu \cdot Y(\lambda t) , \tag{1}$$

where μ is a random variable independent of time.

The variable μ will in general be a function of λ , and will also have probability levels ranging from 0 to 1 .

Let p = Probability of having a Ratio $\frac{Y(t)}{Y(\lambda t)} \leq \mu$.

Due to Monotonicity $\frac{Y(t)}{Y(\lambda t)} \geqslant 1$. Consequently, the zero probability level of the Ratio μ is at $\mu = 1$.

Hence, let us assume that the random ratio μ has a Weibull Distribution

$$p = 1 - 2^{-\left(\frac{\mu - 1}{\theta - 1}\right)^{b}} \tag{2}$$

Where θ and b are undetermined parameters . $\begin{pmatrix} \theta > 1 \\ b > 0 \end{pmatrix}$

The parameter θ is the median value of μ , i. e., the one for which $p = \frac{1}{2}$.

In order to arrive at a reasonable result for the median value of the ratio μ , let us differentiate (1) with respect to time . Thus ,

$$Y'(t) = \mu\lambda \cdot Y'(\lambda t) \tag{3}$$

Because the function $Y(t)$ is monotone increasing , it follows that

$$Y'(t) \geqslant 0 \quad \text{and that} \quad Y'(\lambda t) \geqslant 0 .$$

It is reasonable to expect that 50% of the time the slope at time t will exceed the slope at a previous time λt , and that 50% of the time the slope at time t will be less than the slope at a previous time λt . Therefore , we assume

$$\text{MEDIAN VALUE OF } \mu\lambda = 1 \tag{4}$$

$$\therefore \text{ MEDIAN VALUE OF } \mu = \frac{1}{\lambda} = \theta . \tag{5}$$

Substituting (5) into (2) , we obtain

$$p = 1 - 2^{-\left(\frac{\mu - 1}{\frac{1}{\lambda} - 1}\right)^{b}} \tag{6}$$

Solving (6) for μ :

$$\mu = 1 + \left(\frac{1}{\lambda} - 1\right)\left(\log_2 \frac{1}{1-p}\right)^{\frac{1}{b}} \tag{7}$$

III : DISCUSSION

Upon examining (7) we see that μ is a function of the probability level p and of the value of λ . Therefore, it is appropriate to replace μ by a symbol indicating this functional relationship . Thus ,

$$\mu_p = 1 + \left(\frac{1}{\lambda} - 1\right)\left(\log_2 \frac{1}{1-p}\right)^{\frac{1}{b}} \tag{7a}$$

From this relation it follows that the Median Value of the Random Ratio μ at the value λ is

$$\mu_{.5}(\lambda) = 1 + \frac{1}{\lambda} - 1 = \frac{1}{\lambda} .$$

Hence , the Median Predicted Value of $Y(t)$ from a given value $Y(\lambda t)$ becomes

$$Y(t)_{\text{predicted median}} = \frac{1}{\lambda} Y(\lambda t) . \tag{8}$$

In other words , the Median Value of a Monotone Increasing Function (such as cumulative sales) at time t is twice its value at time $t/2$, three times its value at $t/3$, etc. , which is a reasonable conclusion when the ratio μ is a completely random variable .

IV : CONCLUSION

The Random Ratio Technique herein developed reduces to an assumption of a median condition consisting of a linear increase in a monotone increasing function Y(t) with respect to time. The amount of possible deviation from such a uniform rate of increase will be reflected in the value of the parameter b of the Weibull Distribution assumed for the Random Ration $\mu_p(\lambda)$.

Thus, we have a prediction formula

$$Y(t) = \mu_p(\lambda) \cdot Y(\lambda t) \tag{1a}$$

Where

$$\mu_p(\lambda) = 1 + \left(\frac{1}{\lambda} - 1\right)\left(\log_2 \frac{1}{1-p}\right)^{\frac{1}{b}} \tag{7a}$$

The Probability Level p at which a prediction is made will be called the PREDICTION POLICY. Thus, if a .5 POLICY is in effect, it is understood that the Median Value $\mu_{.5}(\lambda)$ is being used. Significant growth in the value of Y(t) over and above a linear increase will necessitate a p - POLICY with $p > .5$. On the other hand, a slowing down of the growth of Y(t) to less than a linear growth will necessitate a p-POLICY with $p < .5$.

EXAMPLE OF RANDOM RATIO PREDICTION

The cumulative sales of a certain product over the first four months of production is as follows

Cumulative Months	Cumulative Sales
1	10,005 units
2	21,706 units
3	31,921 units
4	40,950 units

QUESTION : What is the predicted Cumulative Sales after 12 cumulative months ?

SOLUTION : There are 6 Sales Ratios. These are

$$ {}_2\mu_1 = \frac{\text{Month 2}}{\text{Month 1}} = \frac{21,706}{10,005} = 2.16952 $$

$$ {}_3\mu_1 = \frac{\text{Month 3}}{\text{Month 1}} = \frac{31,921}{10,005} = 3.19050 $$

$$ {}_4\mu_1 = \frac{\text{Month 4}}{\text{Month 1}} = \frac{40,950}{10,005} = 4.09295 $$

$$ {}_3\mu_2 = \frac{\text{Month 3}}{\text{Month 2}} = \frac{31,921}{21,706} = 1.47061 $$

$$ {}_4\mu_2 = \frac{\text{Month 4}}{\text{Month 2}} = \frac{40,950}{21,706} = 1.88658 $$

$$ {}_4\mu_3 = \frac{\text{Month 4}}{\text{Month 3}} = \frac{40,950}{31,921} = 1.28285 $$

These 6 Sales Ratios have the following values of λ and p associated with them :

Value of Sales Ratio	Subscript Ratio (λ)	p Calculated From (6)
${}_2\mu_1$ = 2.16952	1/2	.61251
${}_3\mu_1$ = 3.19050	1/3	.56460
${}_4\mu_1$ = 4.09295	1/4	.52134
${}_3\mu_2$ = 1.47061	2/3	.45885
${}_4\mu_2$ = 1.88658	1/2	.42006
${}_4\mu_3$ = 1.28285	3/4	.39292

NOTE : The p's are calculated from (6) assuming b = 2 . From experience with actual data the value of b must be known or determinable. It need not be 2 , as here assumed .

$$p_{ave.} = \frac{.61251 + .56460 + .52134 + .45885 + .42006 + .39292}{6}$$

$$p_{ave.} = .49505$$

Now , taking 12 months = 3 x (4 months) , we predict (using λ = 1/3 and $p_{ave.}$ = .49505)

That

$$\text{12 MO. SALES TOTAL} = Y(12) = \mu \cdot (\text{4 MO. SALES TOTAL})$$

Where , $$\mu = 1 + \left(\frac{1}{\lambda} - 1\right)\left(\log_2 \frac{1}{1-p_{ave.}}\right)^{\frac{1}{b}} \qquad \text{(Eqn 7)}$$

i. e., $$\mu = 1 + (3 - 1)\left(\log_2 \frac{1}{1 - .49505}\right)^{1/2} = 2.98573$$

Thus , $$Y(12) = 2.98573\ Y(4) = 2.98573\ (40,950)$$

$$Y(12) = 122,266 \text{ units} \quad \text{(ANSWER)}$$