Statistical Bulletin
Reliability & Variation Research

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THE THEORY OF EVIDENCE

Let H denote some hypothesis which is being investigated for evidence of its truth.

Let C = confidence (probability) that H is true

Then 1 - C = confidence (probability) that H is false

We next define the entropy of H as the natural logarithm of the reciprocal of its survival probability (C being its survival probability).

Thus, E(H) = Entropy of H = $\ln(1/C)$.

Now, let $\overline{H}$ be some competing (alternative) hypothesis (i.e., non-H), such that PROB. (H is true) + PROB. ($\overline{H}$ is true) = 1.

Then,

1 - C = Confidence (probability) that $\overline{H}$ is true (i.e., that H is false).

Furthermore,

$$E(\overline{H}) = \text{Entropy of } \overline{H} = \ln\left(\frac{1}{1 - C}\right).$$

If $C > 1 - C$, it follows that

$$\ln(1/C) < \ln\left(\frac{1}{1 - C}\right)$$

or $E(H) < E(\overline{H})$

We now define the <u>evidence</u> in favor of H (as against $\overline{H}$) as the difference $E(\overline{H}) - E(H)$.

Thus, $Ev(H) = E(\overline{H}) - E(H)$

$$\text{or, } Ev(H) = \ln\left(\frac{1}{1 - C}\right) - \ln(1/C)$$

$$\text{or, } Ev(H) = -\ln(1 - C) + \ln(C)$$

$$\text{or, } Ev(H) = \ln\left(\frac{C}{1 - C}\right).$$

We call $\left(\frac{C}{1 - C}\right)$ the <u>ODDS RATIO</u> in favor of H (as against $\overline{H}$).

Hence, the <u>evidence</u> in favor of any hypothesis H is the natural logarithm of the <u>ODDS RATIO</u> in favor of that same hypothesis.

USE OF THE THEORY IN RELIABILITY TESTING

Suppose a product must be given several tests which are separated due to limitations in time, test facilities, manpower, or economic considerations. For example, one sample of production bearings may have been tested last week and another sample this week. By using the theory of evidence we can combine the separate evidences of last week and this week into a total evidence that production bearings are reliable to some target.

NUMBERICAL EXAMPLE:

WEEK NO.	NO. OF BRGS. TESTED	ESTIMATED WEIBULL PARAMETERS			CONFIDENCE THAT B_{10} Life $\geq$ 100 HR.
		α	b	θ	
1	8	0	1.55	950 hrs.	C_1 = .821
2	10	0	1.68	890 hrs.	C_2 = .864

For WEEK NO. 1 the evidence that B_{10} Life $\geq$ 100 hrs. is

$$\ln\left(\frac{C_1}{1 - C_1}\right) = \ln(.821/.179) = \ln(4.587) = 1.52323.$$

For WEEK NO. 2 the evidence that B_{10} Life $\geq$ 100 hrs. is

$$\ln\left(\frac{C_2}{1 - C_2}\right) = \ln(.864/.136) = \ln(6.353) = 1.84892.$$

Thus, the TOTAL EVIDENCE that B_{10} Life $\geq$ 100 hrs. is

$$1.52323 + 1.84892 = 3.37215.$$

Let $\hat{C}$ = Resultant confidence that B_{10} Life $\geq$ 100 hrs.

Then $$\ln\left(\frac{\hat{C}}{1 - \hat{C}}\right) = 3.37215$$

$$\therefore \frac{\hat{C}}{1 - \hat{C}} = \text{Exp}(3.37215) = 29.142, \text{ or } \hat{C} = .967$$

This value of the resultant confidence is the same as that which is obtained by the SUPERPOSITION FORMULA $\hat{C} = \frac{C_1 C_2}{C_1 C_2 + (1 - C_1)(1 - C_2)}$.

Putting $C_1 = .821$ and $C_2 = .864$, this gives

$$\hat{C} = \frac{(.821)(.864)}{(.821)(.864) + (.179)(.136)} = \frac{.709344}{.709344 + .024344} = \frac{.709344}{.733688} = .967.$$

Thus, each week's testing by itself did not demonstrate 90% confidence but the two weeks superimposed brought the confidence up to 96.7%. Thus, the theory of evidence is very useful in this type of situation. This is one basic reason why small sample testing done repeatedly has been so successful in American industry. This procedure yields more confidence in the end than pooling the two weeks of data into a sample of size 18.

The reader might wonder where we got the weekly confidence indices $C_1 = .821$ and $C_2 = .864$ in the first place. These were obtained from a special DRI computer program known as BQCONF, which is one among the many computer programs which we have in our library. It simply requires the user to supply the following input data:

(1) Quantile Level (.1 in this case, for B_{10} Life)

(2) Target (100 hours in this case)

(3) Min. Life (0 hours in this case)

(4) Weibull Slope (1.55 for Week #1 and 1.68 for Week #2)

(5) Char. Life (950 hrs.for Week #1 & 890 hrs.for Week #2)

(6) Sample Size (8 for Week #1 and 10 for Week #2)

THIS ISSUE'S INTERESTING QUESTION AND ANSWER

QUESTION: What is the mathematical formula L(x) for the left boundary of the 90% confidence band around a Weibull function F(x) as derived from a sample of N failures?

ANSWER:

$$L(x) = 1 - \left[1 - F(x)\right]^{\mu_x}$$

where $$\mu_x = \frac{\text{LOG}\left(1 - .95^{\frac{1}{j_x^{A_x}}}\right)}{\text{LOG}\left(1 - .5^{\frac{1}{j_x^{A_x}}}\right)}$$

where $$j_x = .3 + (N + .4)\,F(x)$$

and $$A_x = 1 + \frac{.45\,N^{.57}\,(j_x - 1)\,(N - j_x)}{(N - 1)^2}$$

Notes, Requests and Thank yous.

Note 1. We would appreciate to have your advice on our next Reliability seminars, Course I and Course II. There does not seem to be time to arrange them yet this year. The last seminars (this year) were held the week of March 15, Course I and the week of April 19, Course II. Should we attempt to offer the seminars earlier like January and February, 1972?

Note 2. Do you have strong preferences to hold the Seminars elsewhere than Detroit suburb of Southfield?

Note 3. We will strive to make the Statistical Bulletin a foremost publication in its field. We would welcome your assistance in this undertaking: your guidance as to content, style, format, problems, questions, etc. It is our belief that we will succeed in this endeavor. Not every statistical publication has Leonard G. Johnson as editor. Leonard has the experience, the ability and is highly creative.

New trends will become apparent in the next issue of the Bulletin.

Note 4. Many thank yous. You and we are aware that laboring in the vineyard of statistical (Weibull) applications to the problems of the mammoth American industry, is highly rewarding to the participants. Many are collecting increased rewards for having taken part in our educational programs.

May we hear from you.

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