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## THE THEORY OF CORROBORATION

Let H = Some Hypothesis

Let P(H) = Initial Probability of Hypothesis (Before Data)

Let P(H if Data) = F

= Final Probability of Hypothesis (After Data)

Let C = Corroboration (Corr.)

## TABLE OF SPECIAL VALUES

| I = Initial Probability | F = Final Probability | C = Corroboration |
|---|---|---|
| P | 1 | 1 |
| P | P | 0 |
| P | 0 | -1 |

## ANALYTICAL FORMULA

$$\text{CORROBORATION} = C = \frac{F - I}{I + F - 2IF}$$

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## CORROBORATION EXAMPLE

QUESTION: If the initial probability of hypothesis H is .7, and if the final probability after data is .9, what is the degree of corroboration of the hypothesis by the data?

## SOLUTION

$$C = \frac{F - I}{I + F - 2IF}$$

$$= \frac{.9 - .7}{.9 + .7 - 2(.9)(.7)}$$

$$= \frac{.2}{1.6 - 1.26}$$

$$= \frac{.2}{.34}$$

$$= .588 \quad \text{(Ans.)}$$

## CORROBORATION IN TERMS OF SUPERPOSITION

Let $I \oplus F = \dfrac{I F}{I F + (1 - I)(1 - F)}$

( NOTE : $\oplus$ is the SUPERPOSITION symbol . )

Then

$$C = \frac{F - I}{I + F - 2 I F} = \frac{F - I}{1 - \left[ I F + (1 - I)(1 - F) \right]}$$

or

$$C = \frac{F - I}{1 - \dfrac{I F}{I \oplus F}}$$

From the example on page 2 :

$$I \oplus F = .7 \oplus .9 = \frac{.63}{.66} = \frac{21}{22} .$$

$$\therefore \quad C = \frac{.9 - .7}{1 - .63 \left( \dfrac{.66}{.63} \right)} = \frac{.20}{.34} = \underline{\underline{.588}}$$

( This is the same answer as that obtained on page 2 . )

## SUPERPOSITION OF CORROBORATIONS

If after $DATA_1$ the corroboration is

$$C_1 = \frac{F_1 - I}{F_1 + I - 2F_1 I}$$

(This is going from $P(H) = I$ to $P(H \text{ if } DATA_1) = F_1$ .)

Then, if after $DATA_2$, the additional corroboration is

$$C_2 = \frac{F_2 - F_1}{F_1 + F_2 - 2F_1 F_2} \qquad \left( \begin{array}{c} \text{Going from} \\ P(H \text{ if } DATA_1) = F_1 \\ \text{to} \\ P(H \text{ if } DATA_2) = F_2 \end{array} \right)$$

We conclude that the RESULTANT CORROBORATION $\hat{C}$ is

$$\hat{C} = \frac{F_2 - I}{F_2 + I - 2F_2 I}$$

or

$$\boxed{\hat{C} = \frac{C_1 + C_2}{1 + C_1 C_2}}$$

(This is the SUPERPOSITION FORMULA for TWO corroborations.)

## CORROBORATION IN TERMS OF PRIOR AND POSTERIOR PROBABILITIES

Let $H_1$ and $H_2$ be alternate hypotheses. Then, we have the following table:

| | Hyp. $H_1$ | Hyp. $H_2$ |
|---|---|---|
| Prior Probability | $C_1$ | $1 - C_1$ |
| Posterior Probability | $\hat{C}$ | $1 - \hat{C}$ |
| Corroboration | $K$ | $-K$ |

The analytical formula for the corroboration $K$ (of hypothesis $H_1$) is

$$K = \frac{\hat{C} - C_1}{C_1 + \hat{C} - 2 C_1 \hat{C}}$$

THEOREM: If we desire a corroboration $K$ for $H_1$, then the posterior probability $\hat{C}$ must be

$$\hat{C} = C_1 \left( \frac{1 + K}{1 - K + 2 C_1 K} \right)$$

For $C_1 = .5$ and $K = .9$ this becomes $\hat{C} = .5 \left( \frac{1 + .9}{1 - .9 + .9} \right) = .95$

Thus, the posterior probability must be 95 % in order to yield 90 % corroboration in this particular case.

## GROUP THEORY OF CORROBORATION

$C_1$ = 1st corroboration

$C_2$ = 2nd corroboration

$\hat{C}$ = Combined Corroboration

$$\hat{C} = \frac{C_1 + C_2}{1 + C_1C_2} = C_1 \oplus C_2$$

Let $C_3$ = 3rd corroboration

Then, the TOTAL COMBINED CORROBORATION of all three corroborations is

$$\hat{\hat{C}} = \hat{C} \oplus C_3 = \frac{\hat{C} + C_3}{1 + \hat{C}C_3}$$

or

$$\hat{\hat{C}} = \frac{C_1 + C_2 + C_3 + C_1C_2C_3}{1 + C_1C_2 + C_1C_3 + C_2C_3}$$

$$= (C_1 \oplus C_2) \oplus C_3$$

$$= C_1 \oplus (C_2 \oplus C_3)$$

$\therefore$ The ASSOCIATIVE LAW holds.

O is the IDENTITY ELEMENT.

The INVERSE of CORROBORATION C is CORROBORATION - C.

## SUPERPOSITION OF 4 CORROBORATIONS

$$C_1 \oplus C_2 \oplus C_3 \oplus C_4 = \frac{C_1+C_2+C_3+C_4+C_1C_2C_3+C_1C_2C_4+C_1C_3C_4+C_2C_3C_4}{1+C_1C_2+C_1C_3+C_1C_4+C_2C_3+C_2C_4+C_3C_4+C_1C_2C_3C_4}$$

In general, when we superimpose n corroborations, we find that

$$\text{RESULTANT CORROBORATION} = \frac{\text{SUM of ALL 1's} + \text{SUM of ALL 3's}}{1 + \text{SUM of ALL 2's} + \text{SUM of ALL 4's}}$$

**THEOREM:** If n successive confidence numbers $C_1, C_2, C_3, C_4, \ldots\ldots C_n$ (that II > I) are observed in n comparison tests, the resultant corroboration for the hypothesis that (II > I) is

$$K^* = 2\,\hat{C}_{234\ldots n} - 1,$$

where $\hat{C}_{234\ldots n}$ is the resultant confidence obtained by superimposing the (n - 1) confidence numbers $C_2, C_3, C_4, \ldots\ldots C_n$.

## PROOF

$$K^* = \frac{\hat{C}_{123\ldots n} - C_1}{\hat{C}_{123\ldots n} + C_1 - 2C_1\hat{C}_{123\ldots n}}$$

This can be shown to equal $2\,\hat{C}_{234\ldots n} - 1$

$$= 2\left[\text{RESULTANT of LAST } (n-1)\ C\text{'s}\right] - 1.$$

## CORROBORATION AS A FUNCTION OF LIKELIHOOD

Let $H_1$ and $H_2$ be two alternate hypotheses. Then, we have the following tabulation:

| $H_1$ | $H_2$ |
|---|---|
| $P(H_1) = C_1$ | $P(H_2) = 1 - C_1$ |
| $P(\text{DATA if } H_1) = L_1 = \text{Likelihood}$ | $P(\text{DATA if } H_2) = L_2$ |
| Let $C_2 = \dfrac{L_1}{L_1 + L_2} =$ Likelihood Probability | $1 - C_2 = \dfrac{L_2}{L_1 + L_2}$ |
| $P(\text{DATA and } H_1) = C_1 C_2$ | $P(\text{DATA and } H_2) = (1 - C_1)(1 - C_2)$ |
| $P(H_1 \text{ if DATA}) = \dfrac{C_1 C_2}{C_1 C_2 + (1-C_1)(1-C_2)}$ | $P(H_2 \text{ if DATA}) = \dfrac{(1-C_1)(1-C_2)}{C_1 C_2 + (1-C_1)(1-C_2)}$ |
| $= \hat{C}$ | $= 1 - \hat{C}$ |
| $= C_1 \oplus \left( \dfrac{L_1}{L_1 + L_2} \right)$ | $= (1 - C_1) \oplus \left( \dfrac{L_2}{L_1 + L_2} \right)$ |
| $= \dfrac{C_1 L_1}{C_1 L_1 + (1 - C_1) L_2}$ | $= \dfrac{(1 - C_1) L_2}{C_1 L_1 + (1 - C_1) L_2}$ |

Thus, it can be seen that the posterior probability is found by superposition of the prior probability and the likelihood probability.

THE CORROBORATION FOR $H_1$ IS $2 C_2 - 1 = K$. THIS IS INDEPENDENT OF ANY PRIOR PROBABILITY $C_1$, i.e., CORROBORATION IS A FUNCTION OF LIKELIHOODS ONLY.