Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
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Vol. 5
Bulletin 8

February, 1976
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THE STATISTICS OF PROTOTYPE TESTING AND LEAD TIME DETERMINATION

QUESTION No. 1

What should be the MTBF (Characteristic Life) goal for a machine ?

ANSWER TO QUESTION No. 1

On the average, a newly designed machine should be reliable enough to permit it to run at least one full season without failure, plus an extra assurance factor. We reason thus because once the season of use is over there is time for necessary replacements or repairs.

EXAMPLE

A machine is used for one month each year during harvest time for 8 hours a day (for 20 working days to the month). This tells us that the MTBF (Characteristic Life) should be $\theta_{seasonal} = 8 \text{ hrs.} \times 20 = 160 \text{ hrs.}$ Putting in 40 hours extra for an assurance factor (i.e., 25% of 160 hrs.) will raise the MTBF (Characteristic Life) goal to 200 hours for the machine.

DESIGN GOAL WITH SPECIFIED RELIABILITY

Designing the machine so as to have RELIABILITY R of going through one season of use without failure will necessitate an MTBF goal given by the formula

$$\left(\text{MTBF}\right)_{\substack{\text{Rel.}\\ R}} = \frac{\theta_{seasonal}}{\ln \frac{1}{R}}$$

(This formula assumes that the Weibull Slope is unity.)

Published 8 times a year by the Detroit Research Institute. Subscription Rate: One year $32.00. Quantity rates on request.
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Detroit Research Insitute, 21900 Greenfield Road, Oak Park, Michigan 48237

In case the Weibull Slope for the entire machine is known to have a specific value (b), then the required CHARACTERISTIC LIFE goal to assure RELIABILITY R of a failure-free season is

$$\theta_R = \frac{\theta_{seasonal}}{\left(\ln \frac{1}{R}\right)^{1/b}}$$

Thus, for $\theta_{seasonal}$ = 160 hrs, and a Weibull Slope of unity, would require (for 90% Reliability for one season of use) that the MTBF goal is set at

$$\frac{160}{\ln \frac{1}{.9}} = 1519 \text{ hours}$$

For 2 to 1 odds ($66\,^2/_3$ % Reliability) against a failure in one season of use, the MTBF goal (for unit Weibull Slope) should be

$$\frac{160}{\ln \frac{1}{.66667}} = 395 \text{ hours}$$

If the Weibull Slope is b = 2, then we would have 2 to 1 odds against failures in one season with a design goal of

$$\theta_{Goal} = \frac{160}{\left(\ln \frac{1}{.66666667}\right)^{1/2}} = 252 \text{ hours}$$

THE RELIABILITY FOR A SET GOAL

QUESTION NO. 2 -- Suppose the MTBF goal is set at 200 hours, while the duration of a season is 160 hours. What RELIABILITY level does the set goal of 200 hours represent as far as a failure-free season is concerned ?

ANSWER TO QUESTION NO. 2

Let θ_{goal} = Set Goal

Let $\theta_{seasonal}$ = Duration of one season

Then the RELIABILITY (R) represented by the set goal is given by the formula (for unit Weibull Slope):

$$R = e^{-\frac{\theta_{seasonal}}{\theta_{goal}}}$$

For $\theta_{seasonal}$ = 160 hours, and θ_{goal} 200 hours, this formula yields

$$R = e^{-\frac{160}{200}} = e^{-.8} = .45$$

Thus, the 200 hour MTBF Goal assures 45% Reliability of a failure-free season if the Weibull Slope is unity.

In case the Weibull Slope is known to have the value b:

$$R = e^{-\left(\frac{\theta_{seasonal}}{\theta_{goal}}\right)^b}$$

Thus, for b = 2, the 200 hour design goal represents a seasonal reliability

$$R = e^{-\left(\frac{160}{200}\right)^2} = e^{-.64} = .53$$

i.e., a RELIABILITY of 53% that the machine will be able to go through one season without failure.

<u>QUESTION NO. 3</u>: How many prototype machines should be tested?

ANSWER TO QUESTION NO. 3

Enough machines should be tested to yield at least 25 component failures when all of the machines have been run to the desired MTBF GOAL of the design. This will guarantee that the observed MTBF (Characteristic Life) will be within ± 20% of the true MTBF with about 2 to 1 odds. The table on the next page will serve as a guide to the number of prototype machines needed:

RATIO : $\dfrac{\text{INITIAL OBSERVED MTBF}}{\text{DESIRED MTBF}}$	NUMBER OF PROTOTYPE MACHINES NEEDED
.20	5
.40	10
.60	15
.80	20
1.00	25

QUESTION NO. 4 : How are prototype data to be collected and recorded ?

ANSWER TO QUESTION NO. 4

We keep a record of the time intervals between successive failures on each prototype machine. Since we are interested in the entire machine, the time intervals recorded are from failure to failure, regardless of component or mode. We are not studying one component or one mode of failure, but the entire machine, no matter what the malfunction might be. We run each prototype machine to the design goal θ_{goal}.

After running each prototype machine to θ_{goal}, we will have a collection of time intervals between failures on each machine. For feasibility of analysis, there should be at least 25 such time intervals in the total collection of machines tested (25 in totality --- not 25 per machine).

QUESTION NO. 5 : How are the prototype failure data to be analyzed ?

ANSWER TO QUESTION NO. 5

Construct a WEIBULL PLOT of all the time intervals between successive failures in the same machine for the entire collection of machines. We will then have a collection of T time intervals (each one representing the elapsed time to the first failure or the time from one failure to the next in the same machine).

We use MEDIAN RANKS for a sample size T as the PLOTTING POSITIONS for these time intervals (after the time intervals have been numerically ORDERED from the shortest to the longest).

E XAMPLE OF PROTOTYPE FAILURE DATA

Suppose 4 prototype machines are being tested. The design goal has been set at θ_{goal} = 300 hours (while 1 season = 160 hours). We run the 4 machines for 300 hours each. The failure record on the 4 machines is listed below :

ELAPSED HOURS (FROM TIME ZERO) TO OBSERVED FAILURES

MACHINE NO. 1	MACHINE NO. 2	MACHINE NO. 3	MACHINE NO. 4
10 hrs.	25 hrs.	5 hrs.	14 hrs.
35 hrs.	60 hrs.	12 hrs.	30 hrs.
100 hrs.	120 hrs.	48 hrs.	67 hrs.
225 hrs.	153 hrs.	65 hrs.	105 hrs.
250 hrs.	215 hrs.	175 hrs.	188 hrs.
	229 hrs.	198 hrs.	260 hrs.
		240 hrs.	271 hrs.
		288 hrs.	

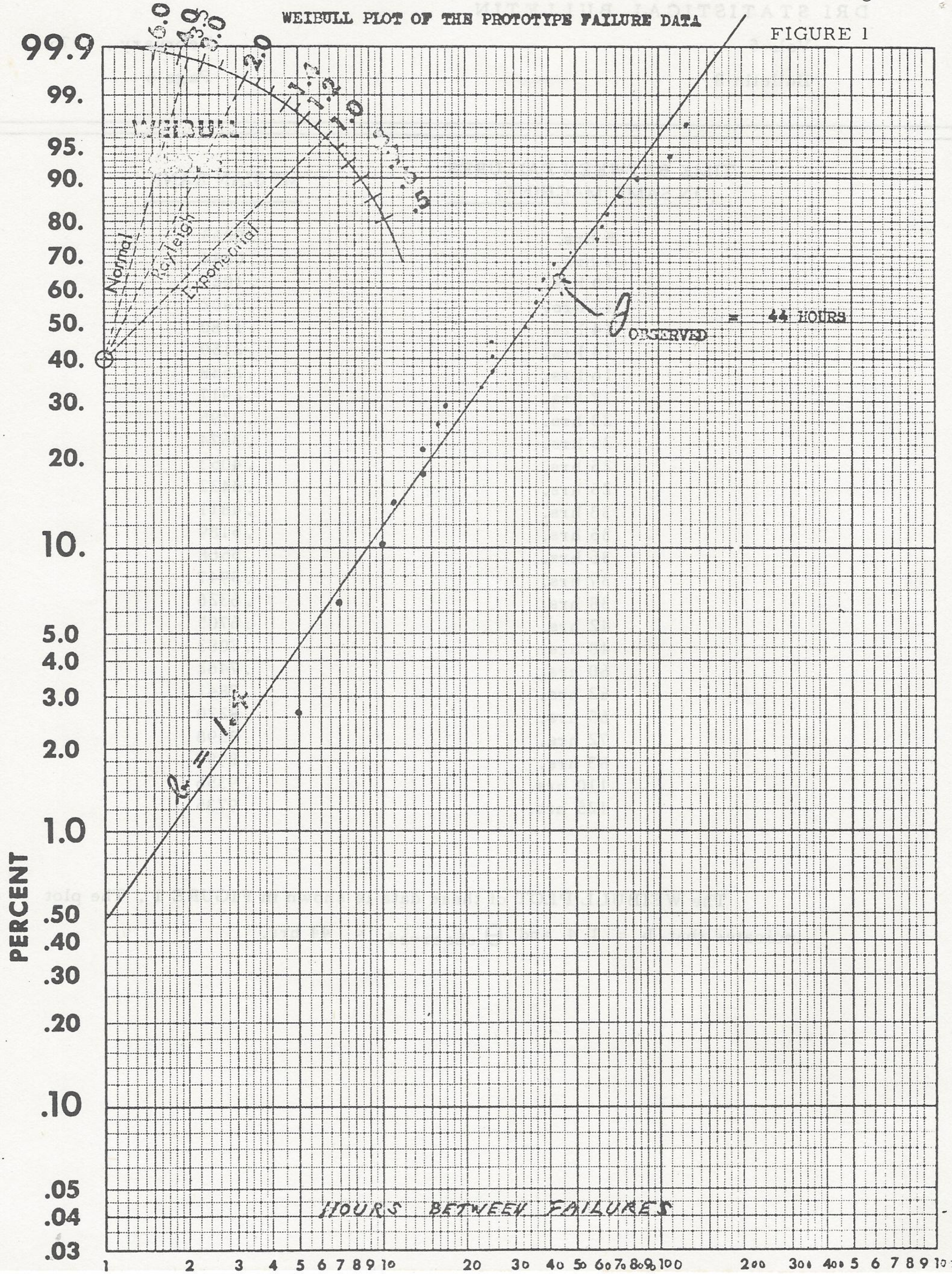
ANALYZING THE PROTOTYPE FAILURE DATA

From the above collection of raw data, we form a table of 26 ORDERED TIME INTERVALS (with MEDIAN RANKS for a sample of 26), as shown on the next page.

TIME BETWEEN FAILURES (ON SAME MACHINE)	MEDIAN RANK
5 hrs.	.0265
7 hrs.	.0644
10 hrs.	.1023
11 hrs.	.1402
14 hrs.	.1780
14 hrs.	.2159
16 hrs.	.2538
17 hrs.	.2917
23 hrs.	.3295
25 hrs.	.3674
25 hrs.	.4053
25 hrs.	.4432
33 hrs.	.4811
35 hrs.	.5189
36 hrs.	.5568
37 hrs.	.5947
38 hrs.	.6326
42 hrs.	.6705
48 hrs.	.7083
60 hrs.	.7462
62 hrs.	.7841
65 hrs.	.8220
72 hrs.	.8598
83 hrs.	.8997
110 hrs.	.9356
125 hrs.	.9735

The WEIBULL PLOT of these data is shown in FIGURE 1 . The plot indicates that b = 1.4 and $\theta_{observed}$ = 44 hrs.

FIGURE 1


WEIBULL PLOT OF THE PROTOTYPE FAILURE DATA
99.9
99.
95.
90.
80.
70.
60.
50.
40.
30.
20.
10.
5.0
4.0
3.0
2.0
1.0
.50
.40
.30
.20
.10
.05
.04
.03
PERCENT
6.0
4.9
3.0
2.0
1.0
.5
Normal
Rayleigh
Exponential
$\theta_{OBSERVED}$ = 44 HOURS
β = 1.7
HOURS BETWEEN FAILURES
1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000

QUESTION NO. 6: How can the Weibull plot of the time intervals between failures tell us the needed machine improvement factor?

ANSWER TO QUESTION NO. 6

From the Weibull plot we find the value of the observed machine characteristic life. This is denoted by $\theta_{observed}$, and is located at the 63.2% level of the Weibull plot. For example, in FIGURE 1, we see that

$$\theta_{observed} = 44 \text{ hours}$$

On the other hand, the value of θ_{goal} was 300 hours.

Therefore, the needed MACHINE LIFE IMPROVEMENT FACTOR is

$$\frac{\theta_{goal}}{\theta_{observed}} = \frac{300}{44} = 6.82$$

Thus, the MACHINE LIFE must be MULTIPLIED by the FACTOR 6.82 to meet the 300 hour goal.

QUESTION NO. 7: What ENGINEERING CHANGES are dictated by the Weibull plot?

ANSWER TO QUESTION NO. 7

Each component that failed in the initial run of the prototype machines to elapsed time θ_{goal} must be improved by the LIFE FACTOR

$$\rho = \frac{\theta_{goal}}{\theta_{observed}}$$

For the machines which produced FIGURE 1, this factor is

$$\rho = 6.82$$

QUESTION NO. 8 : How can the "AS IS" RELIABILITY (for one season) of a machine be determined from a Weibull Plot such as FIGURE 1 ?

ANSWER TO QUESTION NO. 8

The "AS IS" RELIABILITY for 1 season is given by the formula

$$R_{\text{"AS IS"}}(1 \text{ season}) = e^{-\left(\frac{\theta_{seasonal}}{\theta_{observed}}\right)^{b}}$$

where b = Slope of the Weibull plot.

For the example in FIGURE 1 : $R_{\text{"AS IS"}}(160 \text{ hrs.}) = e^{-\left(\frac{160}{44}\right)^{1.4}}$

$= .002255$

QUESTION NO. 9 : If the required improvements are made in all failed components which are found in the initial run to θ_{goal}, what will the "GOOD" RELIABILITY be for 1 season after such improvements ?

ANSWER TO QUESTION NO. 9

$$R_{\text{"GOOD"}}(1 \text{ season}) = e^{-\left(\frac{\theta_{seasonal}}{\theta_{goal}}\right)^{b}}$$

(Compare with QUESTION NO. 2)

For the example in FIGURE 1 : $R_{\text{"GOOD"}}(160 \text{ hrs.}) = e^{-\left(\frac{160}{300}\right)^{1.4}}$

$= .6605$

QUESTION NO. 10 : If a machine's "GOOD" RELIABILITY is too low, what does this indicate ?

ANSWER TO QUESTION NO. 10

A low "GOOD" RELIABILITY simply indicates that the value of θ_{goal} has been chosen too low .

QUESTION NO. 11 : If a "GOOD" RELIABILITY of 99% is desired, how high must Θ_{goal} be set ?

ANSWER TO QUESTION NO. 11

The required goal must be made equal to

$$\Theta_{goal} = = \frac{\Theta_{seasonal}}{\left(\ln \frac{1}{.99}\right)^{1/b}}$$

(b = WEIBULL SLOPE of the plot of the prototype data.)

NOTE : In general, if a "GOOD" RELIABILITY equal to $R_{\text{"GOOD"}}$ is desired, then the required goal is

$$\Theta_{goal} = \frac{\Theta_{seasonal}}{\left(\ln \frac{1}{R_{\text{"GOOD"}}}\right)^{1/b}}$$

QUESTION NO. 12 : What is the relationship between ENGINEERING CHANGE EFFECTIVENESS and LEAD TIME ?

ANSWER TO QUESTION NO. 12

ENGINEERING CHANGE EFFECTIVENESS is defined to be the fraction of failed components in the first run to Θ_{goal} which is eliminated in each RERUN to Θ_{goal}. Thus,

Let R_1 = No. of failures on N machines during RUN # 1 to Θ_{goal}.

Let $R_1 - K$ = No. of failures on N machines during RUN #2 to Θ_{goal} (After the first set of Engineering Changes)

Then, the LEAD TIME (with 50% confidence) required is

$$L_{.50} = \left(1 + \frac{R_1 - N}{K}\right)\Theta_{goal}$$

(See Derivation on page 13)

where,

$$K/R_1 = \text{ENGINEERING CHANGE EFFECTIVENESS} = E$$

Thus,

$$L_{.50} = \Theta_{goal}\left[1 + \frac{1}{E}\left(1 - \frac{N}{R_1}\right)\right]$$

QUESTION NO. 13 : How can we derive a general set of LEAD TIME FORMULAS, ranging from the most optimistic to the most pessimistic ?

ANSWER TO QUESTION NO. 13

We can derive a general set of LEAD TIME FORMULAS by means of a GENERAL THEORY OF ENGINEERING CHANGE EFFECTIVENESS, which allows for possible variations in failure reduction rates per rerun to θ_{goal}, as shown below.

THE GENERAL OUTLOOK ON LEAD TIME

(Reducing the failure count by 1 on each rerun)

1st	θ_{goal}	Hours on N machines :	R_1 failures
2nd	θ_{goal}	Hours on N machines :	$(R_1 - 1)$ failures
3rd	θ_{goal}	Hours on N machines :	$(R_2 - 2)$ failures
.			.
.			.
.			.
.			.
.			.
$(R_1 - N + 1)$th	θ_{goal}	Hours on N machines :	$R_1 - (R_1 - N) = N$ failures

Reduction of failure count by 1 is the slowest rate at which there still is progress on each rerun. Our objective on the MTBF has been realized when N machines have N failures in θ_{goal} hours.

According to this, an upper limit on lead time would be :

$$\text{LEAD TIME} = (R_1 - N + 1)\,\theta_{goal}$$

REDUCING THE FAILURE COUNT BY 2 ON EACH RERUN MADE UP TO Θ_{GOAL}

1st run of Θ_{goal} : R_1 failures

2nd run of Θ_{goal} : $(R_1 - 2)$ failures

3rd run of Θ_{goal} : $(R_1 - 4)$ failures

.
.

ith run of Θ_{goal} : $R_1 - 2(i - 1)$ failures

.
.

Final run of Θ_{goal} : N failures

The index i for the final run is such that $R_1 - 2(i - 1) = N$,

or,
$$i = 1 + \frac{R_1 - N}{2} .$$

Hence, the LEAD TIME $= \left(1 + \frac{R_1 - N}{2}\right)\Theta_{goal} = L_{.50}$.

In case $\frac{R_1 - N}{2}$ is not an interger, round it upward to the next interger.

GENERAL CASE : REDUCING THE FAILURE COUNT BY K ON EACH RERUN TO Θ_{GOAL}

In general if each rerun to Θ_{goal} reduces the failure count by K failures, the LEAD TIME FORMULA is

$$L_{.50} = \left(1 + \frac{R_1 - N}{K}\right)\Theta_{goal} .$$

For confidence levels other than 50%, the formula is (according to this theory)

$$L_C = \left(1 + \frac{R_1 - N}{K}\right)\left(1 + \frac{t_C}{\sqrt{N}}\right)\theta_{goal}.$$

In case $\left(\frac{R_1 - N}{K}\right)$ is not an interger, round it upward to the next interger.

From the equations

$$\begin{cases} L_{.50} = \left(\frac{R_1}{N}\right)\theta_{goal} & \text{(AVERAGE THEORY)} \\ L_{.50} = \left(1 + \frac{R_1 - N}{K}\right)\theta_{goal} & \text{(GENERAL THEORY)} \end{cases}$$

We have, for equality of the $L_{.50}$'s:

$$1 + \frac{R_1 - N}{K} = \frac{R_1}{N}$$

Solving this for K yields $K = N$. Thus, the AVERAGE THEORY, which gave us the formula

$$L_{.50} = \left(\frac{R_1}{N}\right)\theta_{goal},$$

assumes that we reduce the failure count by N on each rerun to θ_{goal}. This is reasonable, if we assume that each component failure which is eliminated gets rid of 1 failure in each machine, and since there are N machines, each FIX (REDESIGN) would reduce the failure count by N on each rerun to θ_{goal}.

QUESTION NO. 14: What is the AVERAGE LEAD TIME and its STANDARD DEVIATION?

ANSWER TO QUESTION NO. 14

We simply construct a NORMAL DISTRIBUTION CURVE whose MEAN is at $\left(\frac{R_1}{N}\right)\theta_{goal}$ and whose lower 3 sigma tail end is at θ_{goal}.

(SEE FIGURE 2) (N = No. of prototype machines
R_1 = No. of failures during first run to θ_{goal})

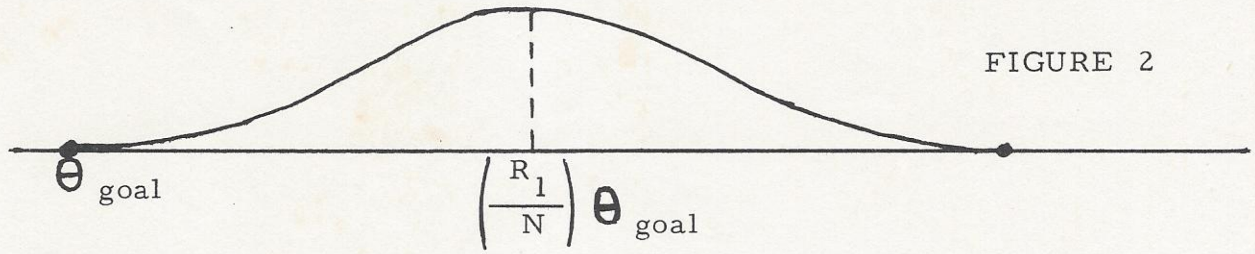


FIGURE 2

From FIGURE 2 :

$$3\sigma = \left(\frac{R_1}{N}\right)\theta_{goal} - \theta_{goal}$$

(NOTE : σ = Standard Deviation of LEAD TIME)

Thus ,

$$\text{Average Lead Time} = L_{AVE.} = \left(\frac{R_1}{N}\right)\theta_{goal}$$

$$\text{Std. Deviation of Lead Time} = \sigma = \frac{1}{3}\left(\frac{R_1}{N} - 1\right)\theta_{goal}$$

Therefore , with 90% confidence, the LEAD TIME will be in the interval

$$\left(\frac{R_1}{N}\right)\theta_{goal} \pm \frac{1.645}{3}\left(\frac{R_1}{N} - 1\right)\theta_{goal}$$

$$= \theta_{goal}\left[\left(\frac{R_1}{N}\right) \pm .5483\left(\frac{R_1}{N} - 1\right)\right]$$

NOTES AND REMINDERS

1. If R_1 turns out to be less than N , then the desired θ_{goal} has been attained , and no further testing is needed on prototype machines.

2. The Lead Time Formulas herein derived do not include any delays or waiting periods for the availability of improved components where they are needed .

3. This analysis assumes that improved components are installed in all of the prototype machines at the beginning of each rerun to θ_{goal}.