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THEORY AND APPLICATION OF THE TABLE OF "AT LEAST" COEFFICIENTS IN SYSTEMS RELIABILITY PROBLEMS

A TYPICAL PROBLEM

Suppose we are given a collection of several items (call it a system), and each of these items has its own specific probability (reliability) of surviving for a certain number of hours of operation. we can then ask such questions as the following:

(1) What is the probability that AT LEAST ONE of the items will survive the required hours of operation ?

(2) What is the probability that AT LEAST TWO of the items will survive the required hours of operation ?

(3) What is the probability that AT LEAST THREE of the items will survive the required hours of operation ?

(4) What is the probability that AT LEAST FOUR of the items will survive the required hours of operation ?

etc. etc. etc.

A SPECIFIC NUMERICAL EXAMPLE

To be specific, let us take a collection of FOUR items with the following individual reliabilities for surviving the required time in service :

ITEM NO.	RELIABILITY TO REQUIRED SERVICE TIME
1	$R_1 = .70$
2	$R_2 = .80$
3	$R_3 = .75$
4	$R_4 = .90$

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Let F_1 = Probability that Item No. 1 fails to last the required service time = $1 - R_1 = 1 - .70 = .30$

Let F_2 = Probability that Item No. 2 fails to last the required service time = $1 - R_2 = 1 - .80 = .20$

Let F_3 = Probability that Item No. 3 fails to last the required service time = $1 - R_3 = 1 - .75 = .25$

Let F_4 = Probability that Item No. 4 fails to last the required service time = $1 - R_4 = 1 - .90 = .10$

The possible outcomes for this collection of four items are as follows :

TABLE I

OUTCOME NO.	DESCRIPTION OF THE OUTCOME
1	all four fail
2	Item No. 1 survives & other three fails
3	Item No. 2 survives & other three fails
4	Item No. 3 survives & other three fail
5	Item No. 4 survives & other three fail
6	Items 1 & 2 survive while 3 & 4 fail
7	Items 1 & 3 survive while 2 & 4 fail
8	Items 1 & 4 survive while 2 & 3 fail
9	Items 2 & 3 survive while 1 & 4 fail
10	Items 2 & 4 survive while 1 & 3 fail
11	Items 3 & 4 survive while 1 & 2 fail
12	Item No. 1 fails & other three survive
13	Item No. 2 fails & other three survive
14	Item No. 3 fails & other three survive
15	Item No. 4 fails & other three survive
16	all four survive

The 16 outcomes on the last page are all the possible outcomes for a collection of 4 items, since a collection of N items can have a total of 2^N outcomes [two outcomes per item (fail or survive)].

For N = 4, this gives us 2^4 = 16 outcomes.

We can now tabulate the probabilities for these 16 outcomes as follows:

TABLE II

OUTCOME NO.	PROBABILITY OF OUTCOME
1	$F_1F_2F_3F_4$ = .0015
2	$R_1F_2F_3F_4$ = .0035
3	$R_2F_1F_3F_4$ = .0060
4	$R_3F_1F_2F_4$ = .0045
5	$R_4F_1F_2F_3$ = .0135
6	$R_1R_2F_3F_4$ = .0140
7	$R_1R_3F_2F_4$ = .0105
8	$R_1R_4F_2F_3$ = .0315
9	$R_2R_3F_1F_4$ = .0180
10	$R_2R_4F_1F_3$ = .0540
11	$R_3R_4F_1F_2$ = .0405
12	$F_1R_2R_3R_4$ = .1620
13	$F_2R_1R_3R_4$ = .0945
14	$F_3R_1R_2R_4$ = .1260
15	$F_4R_1R_2R_3$ = .0420
16	$R_1R_2R_3R_4$ = .3780
	TOTAL = 1.0000

From Table II we can now set that

Prob (at least one survives) = sum of probabilities of outcomes 2 thru 16
= .9985

Prob (at least two survive) = sum of probabilities of outcomes 6 thru 16
= .9710

Prob (at least three survive) = sum of probabilities of outcomes 12 thru 16
= .8025

Prob (at least four survive) = sum of probabilities of outcome 16
= .3780

These same results could have been obtained by using the Table of "AT LEAST" coefficient on the following page .

TABLE III

TABLE OF "AT LEAST" COEFFICIENTS

	$S^{(1)}$	$S^{(2)}$	$S^{(3)}$	$S^{(4)}$	$S^{(5)}$	$S^{(6)}$	$S^{(7)}$	$S^{(8)}$	$S^{(9)}$	$S^{(10)}$
At Least One	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
At Least Two		+1	-2	+3	-4	+5	-6	+7	-8	+9
At Least Three			+1	-3	+6	-10	+15	-21	+28	-36
At Least Four				+1	-4	+10	-20	+35	-56	+84
At Least Five					+1	-5	+15	-35	+70	-126
At Least Six						+1	-6	+21	-56	+126
At Least Seven							+1	-7	+28	-84
At Least Eight								+1	-8	+36
At Least Nine									+1	-9
At Least Ten										+1

$S^{(1)}$ = Sum of R's = $R_1 + R_2 + R_3 + R_4$ = 3.15

$S^{(2)}$ = Sum of all pairs of R's = $R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4$
= 3.71

$S^{(3)}$ = Sum of all Triplets of R's = $R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4$
= 1.9365

$S^{(4)}$ = Sum of all Quadruplets of R's = $R_1R_2R_3R_4$ = .3780

According to Table III :

Prob. (At Least One Survives) $= S^{(1)} - S^{(2)} + S^{(3)} - S^{(4)}$

$= 3.15 - 3.71 + 1.9365 - .3780 = .9985$

Prob. (At Least Two Survive) $= S^{(2)} - 2S^{(3)} + 3S^{(4)}$

$= 3.71 - 2(1.9365) + 3(.3780) = .9710$

Prob. (At Least Three Survive) $= S^{(3)} - 3S^{(4)}$

$= 1.9365 - 3(.3780) = .8025$

Prob. (At Least Four Survive) $= S^{(4)} = .3780$

Thus , the results from Table III are identical to those from Table II.

Table III is easily extended by noting that each column is a set of Binomial Coefficients. Thus, the column headed by $S^{(i)}$ has Binomial Coefficients for the $(i - 1)^{th}$ power of a Binomial with alternate signs (starting with +1 at the top if i is odd, and starting with -1 at the top if i is even). The diagonal always consists of +1's.

For N items, the Table stops with $S^{(N)}$. Numerically, each number in the Table is the sum of the number to its left plus the number directly above the number to the left.