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reliability decision and statistics.

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Page 1

ATTRIBUTE RELIABILITY COMPUTER PROGRAM

I: THE THEORETICAL BASIS OF CLASSICAL ATTRIBUTE SAMPLING CALCULATIONS

Attribute reliability calculations are closely related to sampling plans based on attributes (pass or fail). In any statistically designed sampling plan, the criterion of selection is the so-called OPERATING CHARACTERISTIC CURVE to which we wish the plan to conform. For infinite populations, the mathematical basis of such OPERATING CHARACTERISTIC (O. C.) CURVES becomes the BINOMIAL DISTRIBUTION. For example, if an infinite population has the fraction F defective, and we take a sample of N items from such a population, the probability that all N are good (not defective) is $P = (1 - F)^N = R^N$, where $R = 1 - F$ (i.e., R is the RELIABILITY). Therefore, if a sampling plan requires a sample of N with ZERO DEFECTIVES, the probability of accepting under such a requirement, when the population quality level (fraction defective) is F, is $(1 - F)^N = R^N$. A plan of sample size N with no defectives allowed is called a (0, N) plan.

In like fashion, if a sample of N is to be taken, and it is accepted only if there is no more than 1 defective, the acceptance probability becomes (assuming reliability R): $P = R^N + N R^{N-1}(1 - R) = R^N + N R^{N-1} F$ (for a (1, N) plan).

For a (2, N) plan, i. e., a sampling plan in which we test N and must find no more than 2 defectives in order to accept, the acceptance probability becomes

$$P = R^{N} + NR^{N-1}F + \frac{N(N-1)}{2} R^{N-2}F^{2}$$

Etc. Etc. Etc.

It will be noted that these expressions contain successive terms of the BINOMIAL expansion $(R + F)^{N}$.

II : MODIFICATION OF THE BINOMIAL FOR ATTRIBUTE RELIABILITY ANALYSIS

A common situation in reliability testing is one in which N specimens are run to target X_o (hours, cycles, miles, kilometers, etc.), and the number S which survive to the target is recorded, as well as the number $D = N - S$, which do not survive to the target (and, hence, are defectives). What is desired is an estimate of population reliability to the target X_o. This is done by increasing N to (N + 1), and taking the target X_o to be located at the $(D + 1)^{th}$ ORDER STATISTIC in a sample of (N + 1) lives. Then the probability of having D or fewer in a sample of N below target life X_o (if the reliability is R) is

$$P = R^{N+1} + (N+1)R^{N}F + \frac{(N+1)(N)}{2} R^{N-1}F^{2} + \dots + \frac{(N+1)(N)\dots(N-D+2)}{D!} R^{N-D+1}F^{D}$$

This acceptance probability becomes smaller as R is reduced. We call $C = (1 - P)$ the CONFIDENCE that the reliability is at least R. Thus, reducing R increases the CONFIDENCE that the population reliability is at least R. We can also make the following statements :

P = ACCEPTANCE PROBABILITY (Given R) = 1 - CONFIDENCE (That Reliability $\geq$ R)

C = CONFIDENCE (That Reliability $\geq$ R) = 1 - ACCEPTANCE PROBABILITY (Given R)

III : THE USE OF RANK TABLES IN ATTRIBUTE RELIABILITY ANALYSIS

All attribute reliability questions involving reliability and confidence can be answered by means of rank tables with the aid of the C-RANK THEOREM:

C-RANK THEOREM : If D defectives are observed in N trials , then the reliability $R_C(X_o)$ (with confidence C to target X_o) is equal to

1 - C-rank of the $(D + 1)^{th}$ order statistic in a sample of size (N + 1).

For example , if 3 defectives to a target X_o are observed in 19 trials , then the reliability to the target X_o with 95 % confidence is

$R_{.95}(X_o)$ = 1 - 95 % rank of 4^{th} order statistic in 20.

From a 95 % rank table : 4^{th} in 20 has 95 % rank = . 34366.

Hence , $R_{.95}(X_o)$ = 1 - . 34366 = . 65634 (Ans.)

IV: AN APPROXIMATION FOR RANK TABLES

To find the C-rank of the j^{th} order statistic in N ($.50 \leq C \leq 1$) * :

Define :

$$A = 1 - \frac{1}{9(N - j + 1)} \tag{1}$$

$$B = 1 - \frac{1}{9j} \tag{2}$$

$$\rho = \frac{N - j + 1}{j} \tag{3}$$

Let U_C = Normal Deviate to level $C \geq .50$.

Calculate :

$$F_C = \left[\frac{AB + U_C\sqrt{A^2(1 - B) + B^2(1 - A) - U_C^2(1 - A)(1 - B)}}{A^2 - U_C^2(1 - A)} \right]^3 \tag{4}$$

Then, the C - rank of the j^{th} in N is

$$Z_C(j, N) = \frac{1}{1 + \frac{\rho}{F_C}} \tag{5}$$

NOTE : U_C can be approximated by

$$U_C = -.32795699 + \left(\sqrt{H^3 + G^2} + G\right)^{1/3} - \left(\sqrt{H^3 + G^2} - G\right)^{1/3} \tag{6}$$

where

$$G = \frac{6.78142266}{(1 - C)^{1/4}} - 7.54702358 \tag{7}$$

and

$$H = 1.01610012 \tag{8}$$

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* In case $C < .5$, let $C' = 1 - C$, and calculate the C' rank of the $(N - j + 1)^{th}$ in N.
Then, C-rank of j^{th} in N = 1 - C' - rank of $(N - j + 1)^{th}$ in N.

From (5) we can determine the reliability of an item which has D defectives in T trials by calculating

$$R_C(X_o) = 1 - Z_C(D+1, T+1) \qquad \text{(By the C-rank theorem)}$$

Where $j = D + 1$

and $N = T + 1$

The complete computer program is listed on page 6. The user tells the program to "RUN" . Then the program makes the request :

" GIVE DEFECTIVES , TRIALS , CONFIDENCE" ?

To this the user should answer (for the problem on page 3 , for example):

3 , 19 , .95

and press the RETURN key.

The result will be printed out as shown in the typical printout on page 6.

In case furthur calculations are desired , the user answers the question "ANY MORE INPUT ?" with a 1 (for "YES") , and then waits for a new request of "GIVE DEFECTIVES , TRIALS , CONFIDENCE"? Otherwise , the user answers with a zero (0) for "NO" , and the program terminates.